

# Slope and Proportional Relationships on Graphs

Grade 8 math · Lesson · Standards: 8.EE.B.5, 8.EE.B.6

**Quick answer:** A proportional relationship has the form  $y = kx$ , and its graph is a straight line through the origin whose slope equals the unit rate  $k$ . Slope is rise over run, and it is the same between any two points because slope triangles are similar. The form  $y = mx + b$  adds a  $y$ -intercept  $b$ .

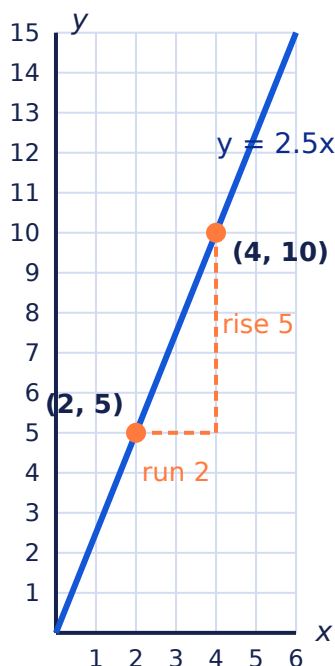
**Standards:** 8.EE.B.5, 8.EE.B.6 · **Level:** Grade 8

## What is a proportional relationship?

### Proportional relationship

Two quantities are proportional when  $y = kx$  for a constant  $k$ . The graph is a straight line through the origin, and  $k$  is the unit rate.

If grapes cost \$2.50 per pound, then  $y = 2.5x$  gives the cost  $y$  of  $x$  pounds.



Cost of grapes: \$2.50 per pound. The slope triangle gives  $\text{rise} \div \text{run} = 5 \div 2 = 2.5$ , the unit rate.

## How do you find the slope of a line?

### Slope

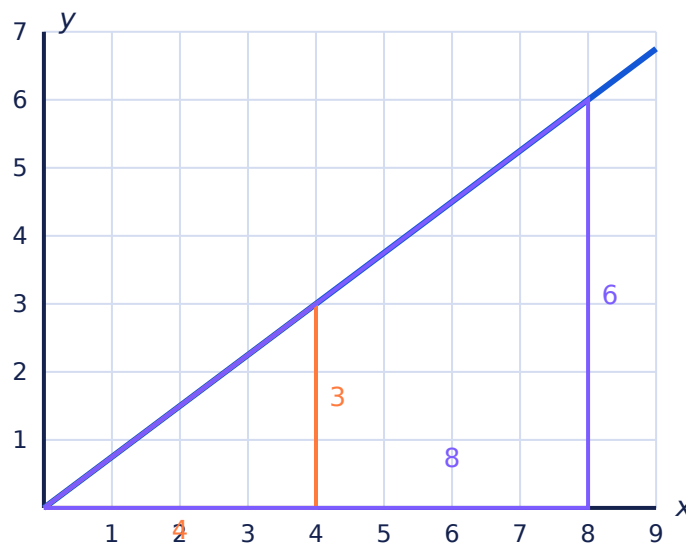
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Slope  $m = \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ . For a proportional relationship, the slope equals the unit rate  $k$ .

| Direction of the line | Slope     |
|-----------------------|-----------|
| Rises left to right   | Positive  |
| Falls left to right   | Negative  |
| Horizontal            | Zero      |
| Vertical              | Undefined |

## Why is the slope the same everywhere on a line?

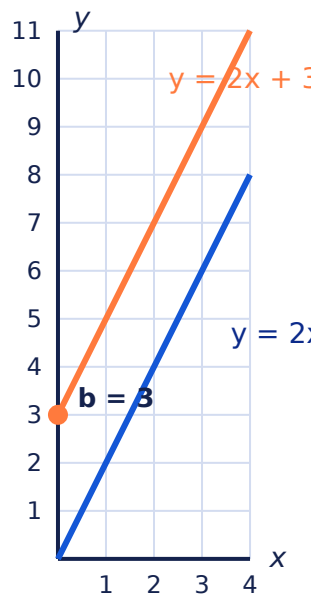
Any two slope triangles on the same line are similar, because their sides are parallel. Similar triangles have equal ratios of rise to run, so the slope never changes.



Both slope triangles are similar, so  $3/4 = 6/8$ : the slope is the same everywhere on the line.

## What does $y = mx + b$ mean?

The slope is  $m$  and the line crosses the  $y$ -axis at  $b$ , the  $y$ -intercept. Setting  $b = 0$  gives the proportional case  $y = mx$ .



Both lines have slope 2. Adding 3 lifts the line to cross the y-axis at  $(0, 3)$ .

### Method: compare two proportional relationships

1. Find the unit rate of each, from a table ( $y \div x$ ), a graph (rise  $\div$  run) or an equation (the number multiplying  $x$ ).
2. Compare the unit rates.
3. The larger unit rate is the steeper line.

### Example

Machine A:  $y = 4.5x$  liters. Machine B: 20 liters in 4 minutes.

B:  $20 \div 4 = 5$  liters per minute. Since  $5 > 4.5$ , **Machine B is faster**.

### Common mistake

Flipping rise and run. Slope is vertical change over horizontal change, never the reverse.

### Mathbot's tip

Beep! Always read a graph from left to right. If the line climbs, the slope is positive; if it drops, the slope is negative.

### Key points to remember

- Proportional:  $y = kx$ ; the graph passes through  $(0, 0)$ .
- Slope  $m = \text{rise} \div \text{run} = (y_2 - y_1) \div (x_2 - x_1)$ .
- In  $y = kx$  the slope is the unit rate  $k$ .

- Similar slope triangles show the slope is constant on a line.
- $y = mx + b$  has slope  $m$  and  $y$ -intercept  $b$ ; it is proportional only when  $b = 0$ .

## **Slope and Proportional Relationships on Graphs: frequently asked questions**

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### **How can you tell if a table shows a proportional relationship?**

Divide  $y$  by  $x$  in every row. If every quotient is the same number, the relationship is proportional and that number is the constant of proportionality.

### **What is the difference between slope and unit rate?**

For a proportional relationship they are equal. The unit rate is the amount of  $y$  per 1 unit of  $x$ , which is exactly the slope of the line through the origin.

### **Why does a proportional graph pass through the origin?**

When  $x = 0$ ,  $y = k \cdot 0 = 0$ . Zero of one quantity matches zero of the other, so the point  $(0, 0)$  is always on the line.

### **What is the slope of a horizontal line?**

It is 0, because the rise is 0 for any run. A vertical line has an undefined slope, since its run is 0 and division by 0 is not allowed.

### **How do similar triangles explain slope?**

Two slope triangles on one line share an angle and have parallel sides, so they are similar. Matching sides are in the same ratio, which makes  $\text{rise} \div \text{run}$  identical for both.