

Solving With Square and Cube Roots

Grade 8 math · Lesson · Standards: 8.EE.A.2

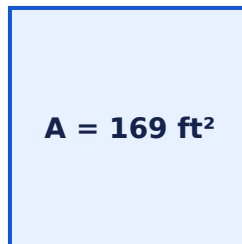
Quick answer: To solve $x^2 = p$, take the square root of both sides: the answers are $\sqrt{p}$ and $-\sqrt{p}$. To solve $x^3 = p$, take the cube root: there is one answer. Perfect squares and cubes give whole-number roots, while roots such as $\sqrt{2}$ are irrational and are estimated.

Standards: 8.EE.A.2 · **Level:** Grade 8

What are square roots and cube roots?

Roots

The **square root** $\sqrt{p}$ is the nonnegative number whose square is p . The **cube root** $\sqrt[3]{p}$ is the number whose cube is p . A **perfect square** or **perfect cube** has a whole-number root.



$$\text{side} = \sqrt{169} = 13 \text{ ft}$$

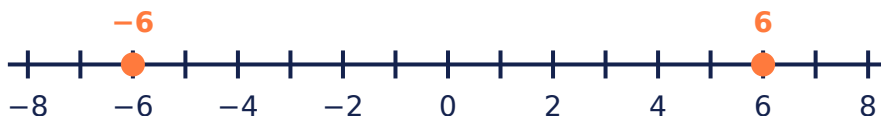
A square with area 169 ft^2 has side $\sqrt{169} = 13 \text{ ft}$.

Number n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	—
12	144	—

How do you solve $x^2 = p$ and $x^3 = p$?

Solving rules (for positive p)

$x^2 = p$ has two solutions, $x = \sqrt{p}$ and $x = -\sqrt{p}$. $x^3 = p$ has exactly one solution, $x = \sqrt[3]{p}$, and it keeps the sign of p .



Both -6 and 6 solve $x^2 = 36$, because $(-6)^2 = (6)^2 = 36$.

Method

1. Isolate the power on one side of the equation.
2. Take the square root (two answers) or the cube root (one answer).
3. If the root is not a whole number, leave it exact and estimate between perfect squares or cubes.
4. In a measurement problem, keep only the positive length.

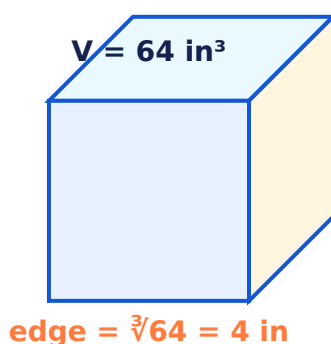
Example: side of a sticker

A square sticker covers 52 cm^2 . Its side s satisfies $s^2 = 52$.

Since $7^2 = 49$ and $8^2 = 64$, $7 < \sqrt{52} < 8$. Testing $7.2^2 = 51.84$ and $7.3^2 = 53.29$ shows the side is about **7.2 cm**.

How do cube roots give an edge length?

The volume of a cube is $V = e^3$, so the edge is $e = \sqrt[3]{V}$. A cube holding 64 in^3 has edge 4 in, because $4 \cdot 4 \cdot 4 = 64$.



A cube with volume 64 in^3 has edge $\sqrt[3]{64} = 4 \text{ in}$.

Roots such as $\sqrt{2}$ and $\sqrt[3]{30}$ are irrational: their decimals never end or repeat, so we write the exact root or an approximation.

Common mistake

Writing $x = 6$ for $x^2 = 36$ and stopping. Also, $x^2 = -16$ has no real solution, because no real number squared is negative; yet $x^3 = -64$ does, namely -4 .

Mathbot's tip

Beep! Memorize the squares up to 15 and the cubes up to 10. They are the landmarks for estimating every other root.

Key points to remember

- $x^2 = p$ (p positive) has two solutions: $\sqrt{p}$ and $-\sqrt{p}$.
- $x^3 = p$ has exactly one solution, $\sqrt[3]{p}$.
- A cube with volume V has edge $\sqrt[3]{V}$; a square with area A has side $\sqrt{A}$.
- Roots of non-perfect squares and cubes are irrational, like $\sqrt{2}$.
- $x^2 = \text{negative number}$ has no real solution; $x^3 = \text{negative number}$ does.

Solving With Square and Cube Roots: frequently asked questions

Why does $x^2 = 49$ have two solutions?

Both 7 and -7 give 49 when squared, since $(-7)(-7) = 49$. We write $x = \pm 7$. In a length problem only the positive value makes sense.

What is the cube root of a negative number?

It is negative. For example $\sqrt[3]{(-125)} = -5$, because $(-5)(-5)(-5) = -125$. Cube roots keep the sign of the number.

Is $\sqrt{2}$ a rational number?

No. $\sqrt{2}$ is irrational, so its decimal 1.41421356... never ends and never repeats. No fraction of integers squares to exactly 2.

How do you estimate a cube root that is not a whole number?

Find the two perfect cubes around the number. Since $64 < 100 < 125$, $\sqrt[3]{100}$ is between 4 and 5. Test tenths, such as $4.6^3 = 97.336$, to refine.

How do you find the side of a square from its area?

Take the square root of the area. A square with area 169 square feet has side $\sqrt{169} = 13$ feet. Use

the positive root only.