

Round Solids: Cylinder, Cone, Sphere Volume

Grade 8 math · Lesson · Standards: 8.G.C.9

Quick answer: A cylinder holds $\pi r^2 h$, where r is the radius of its circular end and h its height. A cone holds one third as much as the matching cylinder, and a sphere holds $(4/3)\pi r^3$. Always use the radius, halving any diameter first, and write the answer in cubic units, with π or with 3.14.

Standards: 8.G.C.9 · **Level:** Grade 8

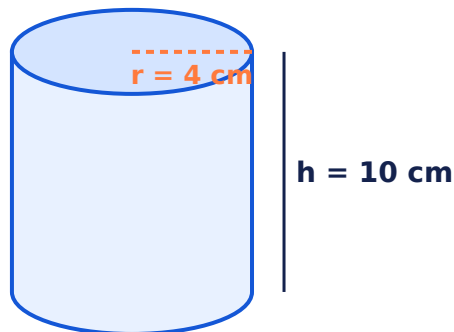
How much can a cylinder hold?

Picture a tall stack of identical flat coins. Every coin covers a circle, and the stack is h units high, so the capacity equals the area of the circular end multiplied by the height.

Cylinder formula

$$V = \pi r^2 h$$

Use the radius of the circular end, not the distance across it.



This cylinder holds $160\pi \text{ cm}^3$.

What about a cone?

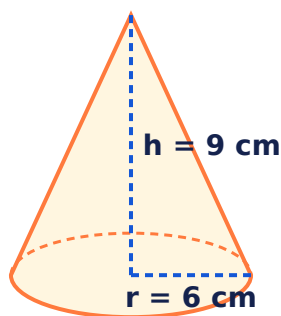
Fill a cone with sand and tip it into a can that has the same opening and the same depth. It takes three trips to fill the can, so a cone is a third of its matching cylinder.

Cone formula

$$V = \frac{1}{3}\pi r^2 h$$

Measure h straight down from the tip to the middle of the base. The slanted edge is longer and

is not used.



This cone holds $108\pi \text{ cm}^3$.

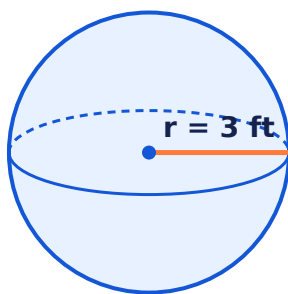
Where does the sphere formula come from?

Archimedes noticed that a ball snugly wrapped in a can, as wide and as tall as the ball, takes up exactly two thirds of the can. Take two thirds of what that can holds and the sphere formula appears.

Sphere formula

$$V = \frac{4}{3}\pi r^3$$

Half a ball, called a hemisphere, holds half of this amount.



This sphere holds $36\pi \text{ ft}^3$.

What steps solve a volume problem?

Method

1. Name the solid and write its formula.
2. Convert a diameter into a radius by halving it.
3. Substitute, cube or square the radius, and multiply.
4. Keep π as a symbol for an exact result, or use 3.14 for a decimal.

5. Finish with cubic units.

Worked example

A soup can is 8 cm across and 11 cm tall. Half of 8 is 4, so the can holds π times 16 times 11, which is $176\pi \text{ cm}^3$. Replacing π with 3.14 gives about 552.64 cm^3 .

Common mistake

Putting the diameter into the formula where the radius belongs makes a cylinder four times too big. Read the given measurement twice before squaring it.

Mathbot's tip

With a cone or a sphere, multiply the whole numbers first, for example $4 \cdot 27 = 108$, and divide by 3 as the last step.

Key points to remember

- Cylinder: $V = \pi r^2 h$.
- Cone: one third of the cylinder that matches its opening and height.
- Sphere: $V = (4/3)\pi r^3$; a hemisphere holds half of that.
- The radius is half of the diameter, and every formula needs it.
- Capacity is measured in cubic units such as cm^3 or ft^3 .

Round Solids: Cylinder, Cone, Sphere Volume: frequently asked questions

Why is a cone one third of a cylinder?

If a cone and a cylinder have the same base and height, three cones of liquid fill the cylinder exactly. This can be shown by pouring, and it gives the factor $1/3$ in the cone formula.

Do I use the radius or the diameter in the formulas?

Always use the radius. If a problem gives the diameter, divide it by 2 first. Using the diameter by mistake makes a cylinder volume four times too large.

What does leaving the answer in terms of π mean?

It means writing π as a symbol, such as $160\pi \text{ cm}^3$, instead of multiplying by 3.14. This is the exact value. Use 3.14 when a problem asks for a decimal approximation.

How do I find the volume of a hemisphere?

A hemisphere is half a sphere, so its volume is $(2/3)\pi r^3$. For a radius of 5 inches, the volume is $(2/3)\pi \cdot 125 = (250/3)\pi$ cubic inches.

How can I find a missing height or radius?

Write the formula with the known volume, substitute the other measurement, and solve. For a cylinder with $V = 225\pi$ and $r = 5$, $25\pi h = 225\pi$ gives $h = 9$.