

Complex Factoring and Sphere Volume

Grade 12 – Precalculus math · Test Answer Key · Standards: N-CN.C.8, G-GMD.A.2

Use the scoring notes for partial credit. Most items award a point for the correct identity or setup and a point for the final factors, zeros or volume.

Standards: N-CN.C.8, G-GMD.A.2 · **Level:** Grade 12 Precalculus

Question 1 Powers of i (2 points)

$38 \div 4$ leaves remainder 2, so $i^{38} = i^2 = -1$. $55 \div 4$ leaves remainder 3, so $i^{55} = i^3 = -i$.

Answer: **-1 and $-i$**

Scoring: 1 point for each power.

Question 2 Factoring Sums of Squares (2 points)

(a) $x^2 + 8^2 = (x + 8i)(x - 8i)$.

(b) $(5x)^2 + 3^2 = (5x + 3i)(5x - 3i)$.

Answer: **(a) $(x + 8i)(x - 8i)$; (b) $(5x + 3i)(5x - 3i)$**

Scoring: 1 point for each part.

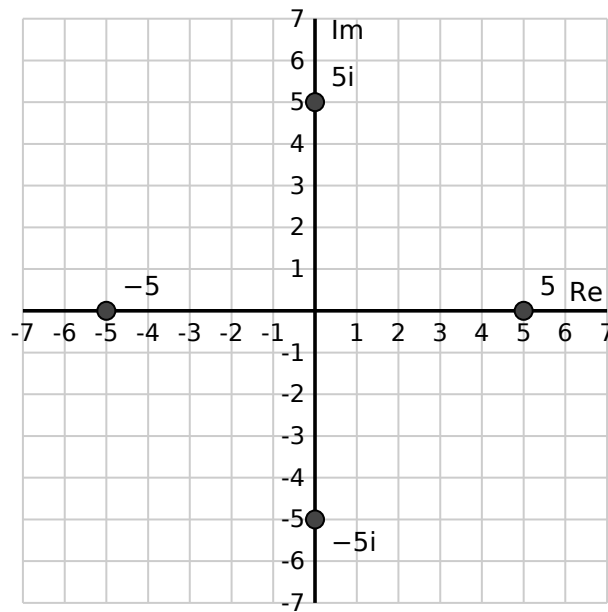
Question 3 A Radical Solution (2 points)

$x^2 = -75$, so $x = \pm\sqrt{75} i = \pm 5\sqrt{3} i$.

Answer: **$x = \pm 5\sqrt{3} i$**

Scoring: 1 point for the setup $x^2 = -75$, 1 point for the simplified solutions.

Question 4 A Fourth-Degree Polynomial (3 points)



The labeled zeros.

(a) $x^4 - 625 = (x^2 - 25)(x^2 + 25) = (x - 5)(x + 5)(x^2 + 25)$.

(b) $(x - 5)(x + 5)(x + 5i)(x - 5i)$.

(c) The points are 5, -5 , $5i$ and $-5i$.

Answer: **$(x - 5)(x + 5)(x + 5i)(x - 5i)$**

Scoring: 1 point for each part.

Question 5 Complex Zeros of a Quadratic (3 points)

(a) $64 - 100 = -36$.

(b) $x = \frac{(8 \pm \sqrt{(-36)})}{2} = \frac{(8 \pm 6i)}{2} = 4 \pm 3i$.

(c) $(x - 4 - 3i)(x - 4 + 3i)$.

Answer: **$x = 4 \pm 3i$**

Scoring: 1 point for each part.

Question 6 Cavalieri's Principle (3 points)

(a) The slice is a disk of radius $\sqrt{(36 - 16)} = \sqrt{20}$, with area 20π .

(b) The slice is a ring with outer radius 6 and inner radius 4, with area $36\pi - 16\pi = 20\pi$.

(c) The slices match at every height and the solids have equal heights, so the two solids have

equal volumes.

Answer: **20π and 20π ; the volumes are equal**

Scoring: 1 point for each part.

Question 7 Volume of a Sphere (3 points)

The radius is 9 cm. $V = \frac{4}{3}\pi(9)^3 = \frac{4}{3}\pi(729) = 972\pi$.

$972\pi \approx 3,053.6$ cubic centimeters.

Answer: **$972\pi \approx 3,053.6 \text{ cm}^3$**

Scoring: 1 point for the radius, 1 point for the formula with values, 1 point for the result.

Question 8 Building a Cubic (2 points)

The conjugate $2 - 3i$ is also a zero. $(x - 2 - 3i)(x - 2 + 3i) = (x - 2)^2 + 9 = x^2 - 4x + 13$.

Multiply by $(x - 1)$: $x^3 - 4x^2 + 13x - x^2 + 4x - 13 = x^3 - 5x^2 + 17x - 13$.

Answer: **$x^3 - 5x^2 + 17x - 13$**

Scoring: 1 point for the conjugate pair factor, 1 point for the product.