

Complex Factoring and Sphere Volume

Grade 12 – Precalculus math · Test · Standards: N-CN.C.8, G-GMD.A.2

Name: _____ Date: _____ Class: _____

This 45-minute test has 8 questions worth 20 points. Show every factoring step, give exact values with π or radicals, and round decimals only when the question asks.

Standards: N-CN.C.8, G-GMD.A.2 · **Level:** Grade 12 Precalculus

Time: 45 minutes · 8 questions · Total: 20 points · Calculator: as allowed by your teacher

Question 1 Powers of i (2 points)

Evaluate i^{38} and i^{55} .

Question 2 Factoring Sums of Squares (2 points)

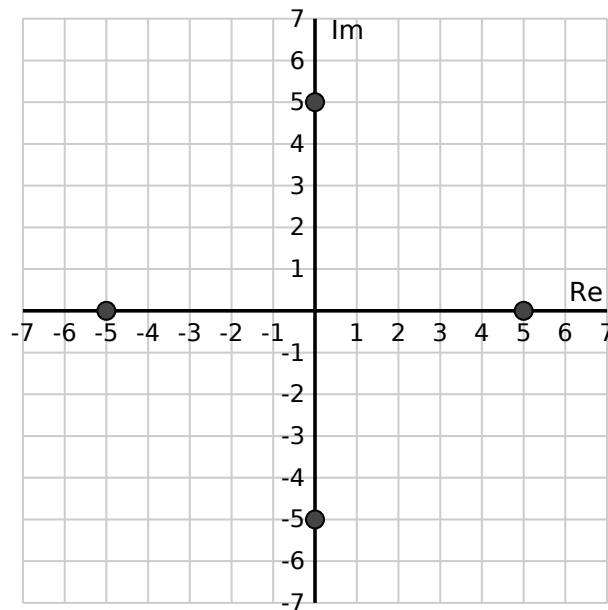
Factor over the complex numbers.

- a. $x^2 + 64$ (1 point)
- b. $25x^2 + 9$ (1 point)

Question 3 A Radical Solution (2 points)

Solve $x^2 + 75 = 0$. Simplify the radical.

Question 4 A Fourth-Degree Polynomial (3 points)



The four zeros of the polynomial are plotted.

The figure plots the four zeros of $x^4 - 625$.

- Factor $x^4 - 625$ completely over the real numbers. (1 point)
- Factor it completely over the complex numbers. (1 point)
- Label the four plotted points with their values. (1 point)

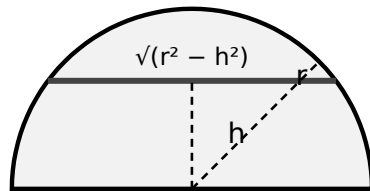
Question 5 Complex Zeros of a Quadratic (3 points)

Consider $x^2 - 8x + 25 = 0$.

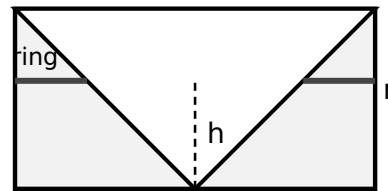
- Find the discriminant. (1 point)
- Solve the equation. (1 point)
- Write the quadratic in factored form over the complex numbers. (1 point)

Question 6 Cavalieri's Principle (3 points)

Hemisphere, radius r



Cylinder minus cone



At every height h both slices have the same area, $\pi(r^2 - h^2)$.

A hemisphere of radius 6 and a cylinder of radius 6 and height 6 with an upside-down cone removed are cut at height 4.

- Find the area of the hemisphere's slice. (1 point)
- Find the area of the cylinder-minus-cone slice. (1 point)
- State what Cavalieri's principle says about the volumes. (1 point)

Question 7 Volume of a Sphere (3 points)

A spherical globe has a diameter of 18 centimeters. Find its volume exactly and to the nearest tenth.

Question 8 Building a Cubic (2 points)

Write a cubic polynomial with real coefficients that has zeros 1 and $2 + 3i$. Give it in standard form.