

Complex Factoring and Sphere Volume

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: N-CN.C.8, G-GMD.A.2

Each answer shows the factoring identity used and checks by multiplying back or substituting. Volume answers give an exact value in terms of π and a rounded decimal.

Standards: N-CN.C.8, G-GMD.A.2 · **Level:** Grade 12 Precalculus

Problem 1 Powers of i

Powers of i repeat every 4. $27 \div 4$ leaves remainder 3, so $i^{27} = i^3 = -i$. $46 \div 4$ leaves remainder 2, so $i^{46} = i^2 = -1$.

Answer: $i^{27} = -i$ and $i^{46} = -1$

Problem 2 Expanding a Product

$$(x + 5i)(x - 5i) = x^2 - 25i^2 = x^2 + 25.$$

Answer: $x^2 + 25$

Problem 3 A Sum of Squares

$$49 = 7^2, \text{ so } x^2 + 49 = x^2 + 7^2 = (x + 7i)(x - 7i).$$

Answer: $(x + 7i)(x - 7i)$

Problem 4 A Non-Square Constant

$$\sqrt{18} = 3\sqrt{2}, \text{ so } x^2 + 18 = (x + 3\sqrt{2}i)(x - 3\sqrt{2}i).$$

Answer: $(x + 3\sqrt{2}i)(x - 3\sqrt{2}i)$

Problem 5 Solving a Sum of Squares

$$x^2 = -36, \text{ so } x = \pm\sqrt{-36} = \pm 6i. \text{ These are the two points on the imaginary axis.}$$

Answer: $x = 6i$ or $x = -6i$

Problem 6 Product of Conjugates

$$(3 + 2i)(3 - 2i) = 9 - 4i^2 = 9 + 4 = 13, \text{ which equals } 3^2 + 2^2 = 13.$$

Answer: **13**

Problem 7 A Leading Coefficient

$$4x^2 + 25 = (2x)^2 + 5^2 = (2x + 5i)(2x - 5i).$$

$$\text{Check: } (2x)^2 - (5i)^2 = 4x^2 + 25.$$

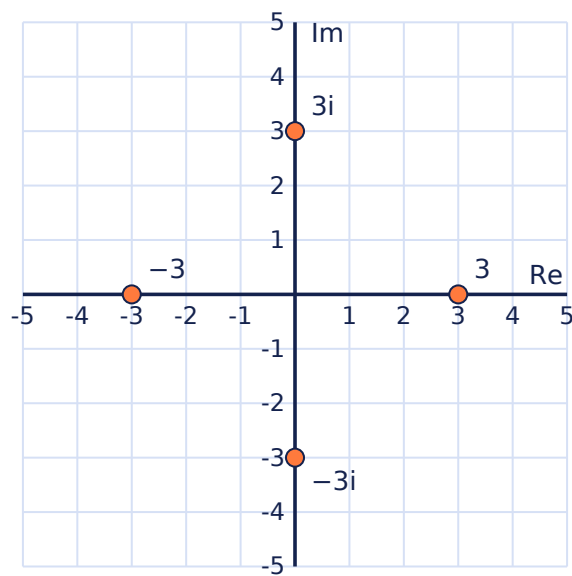
Answer: **$(2x + 5i)(2x - 5i)$**

Problem 8 Two Variables

$$9x^2 + 16y^2 = (3x)^2 + (4y)^2 = (3x + 4yi)(3x - 4yi).$$

Answer: **$(3x + 4yi)(3x - 4yi)$**

Problem 9 A Fourth-Degree Factorization



The four zeros of $x^4 - 81$.

$$x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x + 3i)(x - 3i).$$

The zeros are 3, -3, 3i and -3i.

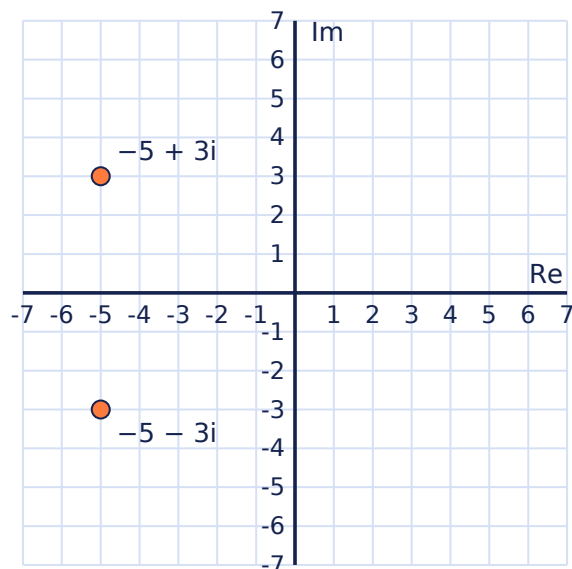
Answer: **$(x - 3)(x + 3)(x + 3i)(x - 3i)$**

Problem 10 Factor Out First

Divide by 2: $x^2 + 49 = 0$, so $x = \pm 7i$. The factored form is $2(x + 7i)(x - 7i)$.

Answer: $x = \pm 7i$; $2(x + 7i)(x - 7i)$

Problem 11 A Quadratic with Complex Zeros



The zeros are a conjugate pair.

The discriminant is $100 - 136 = -36$, so $\sqrt{-36} = 6i$. Then $x = \frac{-10 \pm 6i}{2} = -5 \pm 3i$.

The factored form is $(x + 5 - 3i)(x + 5 + 3i)$.

Check: $(x + 5)^2 - (3i)^2 = x^2 + 10x + 25 + 9$.

Answer: $x = -5 \pm 3i$

Problem 12 Completing the Square

$x^2 - 4x + 4 = -29 + 4$, so $(x - 2)^2 = -25$. Then $x - 2 = \pm 5i$ and $x = 2 \pm 5i$.

Answer: $x = 2 + 5i$ and $x = 2 - 5i$

Problem 13 The General Identity

$(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$.

With $a = 7$ and $b = 4$: $(7 + 4i)(7 - 4i) = 49 + 16 = 65$.

Answer: **65**

Problem 14 Volume of a Sphere

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(216) = 288\pi \text{ cubic inches.}$$

$$288\pi \approx 904.8.$$

Answer: **$288\pi \approx 904.8$ cubic inches**

Problem 15 Cavalieri with a Hemisphere

(a) Hemisphere slice: radius $\sqrt{(25 - 9)} = 4$, area 16π . Cylinder-minus-cone slice: ring with outer radius 5 and inner radius 3, area $25\pi - 9\pi = 16\pi$. The areas match.

(b) The cylinder has volume $\pi(25)(5) = 125\pi$. The cone has volume $\frac{1}{3}\pi(25)(5) = \frac{125\pi}{3}$. The hemisphere is $125\pi - \frac{125\pi}{3} = \frac{250\pi}{3}$, and the sphere is $\frac{500\pi}{3} \approx 523.6$.

Answer: **both slices have area 16π ; hemisphere $\frac{250\pi}{3}$, sphere $\frac{500\pi}{3} \approx 523.6$**

Problem 16 Spherical Water Tank

(a) The radius is 6 ft. $V = \frac{4}{3}\pi(6)^3 = 288\pi \approx 904.8$ cubic feet.

(b) $904.78 \times 7.48 \approx 6,768$ gallons.

Answer: **about 904.8 cubic feet, or about 6,768 gallons**

Problem 17 Cone with a Dome

Cone: $\frac{1}{3}\pi(3^2)(8) = 24\pi$. Hemisphere: $\frac{2}{3}\pi(3^3) = 18\pi$.

Total = $24\pi + 18\pi = 42\pi \approx 131.9$ cubic centimeters.

Answer: **$42\pi \approx 131.9 \text{ cm}^3$**

Problem 18 A Quartic Equation

Treat it as a quadratic in x^2 : $x^4 + 5x^2 - 36 = (x^2 + 9)(x^2 - 4)$, since $9 \cdot (-4) = -36$ and $9 + (-4) = 5$.

Then $x^2 = -9$ gives $x = \pm 3i$, and $x^2 = 4$ gives $x = \pm 2$.

Answer: **$x = 2, -2, 3i, -3i$**

Problem 19 Factoring a Cubic

Group: $x^2(x - 4) + 9(x - 4) = (x - 4)(x^2 + 9)$.

Then $x^2 + 9 = (x + 3i)(x - 3i)$, so the factorization is $(x - 4)(x + 3i)(x - 3i)$.

Answer: **zeros 4, $3i$ and $-3i$**

Problem 20 Building a Polynomial

Complex zeros of a real polynomial come in conjugate pairs, so $3 - i$ is also a zero.

$$(x - 3 - i)(x - 3 + i) = (x - 3)^2 - i^2 = x^2 - 6x + 10.$$

Multiply by $(x - 2)$: $x^3 - 6x^2 + 10x - 2x^2 + 12x - 20 = x^3 - 8x^2 + 22x - 20$.

Check at $x = 2$: $8 - 32 + 44 - 20 = 0$.

Answer: **$x^3 - 8x^2 + 22x - 20$**