

Complex Factoring and Sphere Volume

Grade 12 – Precalculus math · Worksheet · Standards: N-CN.C.8, G-GMD.A.2

Name: _____ Date: _____ Class: _____

These problems extend polynomial identities to the complex numbers (N-CN.C.8), factoring sums of squares and finding complex zeros, and they use Cavalieri's principle to build the sphere volume formula (G-GMD.A.2).

The last problems cover tanks, desserts, quartic equations and polynomials built from given zeros.

Standards: N-CN.C.8, G-GMD.A.2 · **Level:** Grade 12 Precalculus

Problem 1 Powers of i (easy)

Evaluate i^{27} and i^{46} .

Problem 2 Expanding a Product (easy)

Expand $(x + 5i)(x - 5i)$.

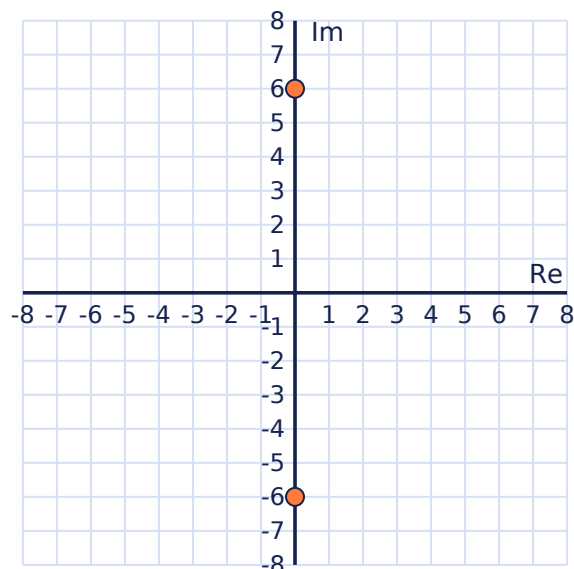
Problem 3 A Sum of Squares (easy)

Factor $x^2 + 49$ over the complex numbers.

Problem 4 A Non-Square Constant (easy)

Factor $x^2 + 18$ over the complex numbers. Simplify the radical.

Problem 5 Solving a Sum of Squares (easy)



Two points on the imaginary axis.

Solve $x^2 + 36 = 0$. The figure shows where the solutions are plotted.

Problem 6 Product of Conjugates (easy)

Compute $(3 + 2i)(3 - 2i)$ and compare it with $3^2 + 2^2$.

Problem 7 A Leading Coefficient (medium)

Factor $4x^2 + 25$ over the complex numbers.

Problem 8 Two Variables (medium)

Factor $9x^2 + 16y^2$ over the complex numbers.

Problem 9 A Fourth-Degree Factorization (medium)

Factor $x^4 - 81$ completely over the complex numbers and list its four zeros.

Problem 10 Factor Out First (medium)

Solve $2x^2 + 98 = 0$ and write the left side in factored form.

Problem 11 A Quadratic with Complex Zeros (medium)

Use the quadratic formula on $x^2 + 10x + 34 = 0$, then write the quadratic in factored form.

Problem 12 Completing the Square (medium)

Find the zeros of $x^2 - 4x + 29$ by completing the square.

Problem 13 The General Identity (medium)

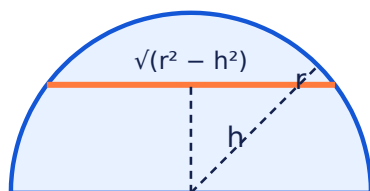
Show that $(a + bi)(a - bi) = a^2 + b^2$, then use it with $a = 7$ and $b = 4$.

Problem 14 Volume of a Sphere (medium)

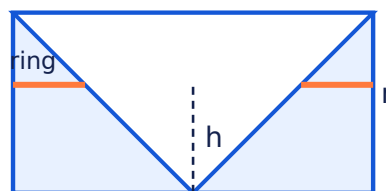
Find the exact volume of a sphere of radius 6 inches and give a decimal rounded to the nearest tenth.

Problem 15 Cavalieri with a Hemisphere (hard)

Hemisphere, radius r



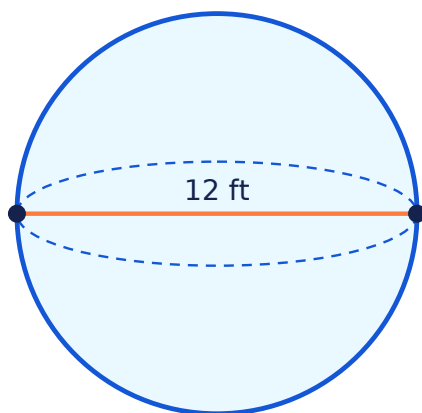
Cylinder minus cone



At every height h both slices have the same area, $\pi(r^2 - h^2)$.

A hemisphere has radius 5. It is compared with a cylinder of radius 5 and height 5 with an upside-down cone removed. (a) Find the area of each solid's slice at height 3. (b) Use the volumes of the cylinder and the cone to find the hemisphere's volume and then the sphere's volume.

Problem 16 Spherical Water Tank (hard)

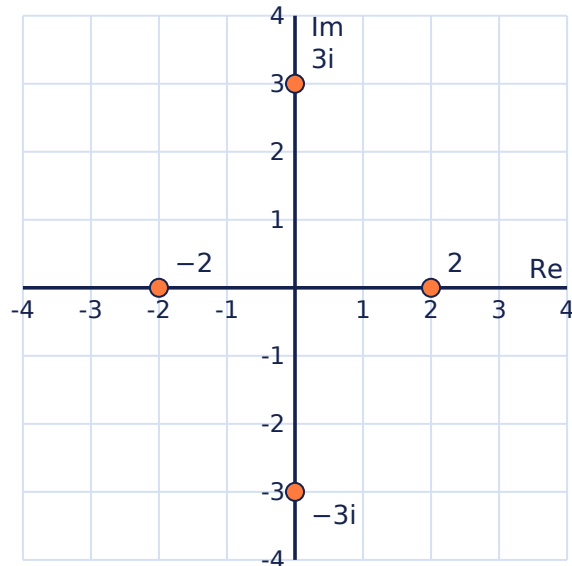


A spherical tank with diameter 12 ft.

A spherical water tank has a diameter of 12 feet. (a) Find its volume in cubic feet. (b) One cubic foot holds about 7.48 gallons. How many gallons does the tank hold, to the nearest gallon?

Problem 17 Cone with a Dome (hard)

A cone-shaped dessert has a base radius of 3 cm and a height of 8 cm. A hemisphere of ice cream with the same radius sits on top. Find the total volume, exactly and to the nearest tenth.

Problem 18 A Quartic Equation (hard)

The four solutions in the complex plane.

Solve $x^4 + 5x^2 - 36 = 0$ over the complex numbers.

Problem 19 Factoring a Cubic (hard)

Factor $x^3 - 4x^2 + 9x - 36$ completely over the complex numbers and list its zeros.

Problem 20 Building a Polynomial (hard)

Find a cubic polynomial with real coefficients that has zeros 2 and $3 + i$. Write it in standard form.