

Polar Form and Powers of Complex Numbers

Grade 12 – Precalculus math · Test Answer Key · Standards: N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3

Use the scoring notes to award partial credit. Most questions give a point for the correct modulus or angle and a point for the final form. Exact values are required wherever the question asks for them.

Standards: N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3 · **Level:** Grade 12 Precalculus

Question 1 Reading the Plane (2 points)

The point is 5 units left and 3 units up, so $z = -5 + 3i$.

(a) $|z| = \sqrt{(25 + 9)} = \sqrt{34}$.

(b) $\bar{z} = -5 - 3i$.

Answer: $\sqrt{34}$; $-5 - 3i$

Scoring: 1 point for each part.

Question 2 Rectangular to Polar (2 points)

(a) $r = \sqrt{(8 + 8)} = 4$.

(b) The reference angle is 45° and the number is in Quadrant II, so $\theta = 135^\circ$, and the polar form is $4(\cos 135^\circ + i \sin 135^\circ)$.

Answer: $r = 4$; $4(\cos 135^\circ + i \sin 135^\circ)$

Scoring: 1 point for each part.

Question 3 Polar to Rectangular (2 points)

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$ and $\sin 210^\circ = -\frac{1}{2}$. So $a = 10 \cdot (-\frac{\sqrt{3}}{2}) = -5\sqrt{3}$ and $b = 10 \cdot (-\frac{1}{2}) = -5$.

Answer: $-5\sqrt{3} - 5i$

Scoring: 1 point for the two values from the unit circle, 1 point for the result.

Question 4 Product and Quotient (2 points)

(a) $6 \cdot 3 = 18$ and $100^\circ + 40^\circ = 140^\circ$, so $18(\cos 140^\circ + i \sin 140^\circ)$.

(b) $6 \div 3 = 2$ and $100^\circ - 40^\circ = 60^\circ$, so $2(\cos 60^\circ + i \sin 60^\circ)$.

Answer: **(a) $18(\cos 140^\circ + i \sin 140^\circ)$; (b) $2(\cos 60^\circ + i \sin 60^\circ)$**

Scoring: 1 point for each part.

Question 5 Distance and Midpoint (2 points)

Difference: $5 - 8i$. Distance = $\sqrt{(25 + 64)} = \sqrt{89} \approx 9.43$.

Midpoint = $\frac{(1 + 6i)}{2} = \frac{1}{2} + 3i$.

Answer: **$\sqrt{89}$ and $\frac{1}{2} + 3i$**

Scoring: 1 point for the distance, 1 point for the midpoint.

Question 6 Using De Moivre (3 points)

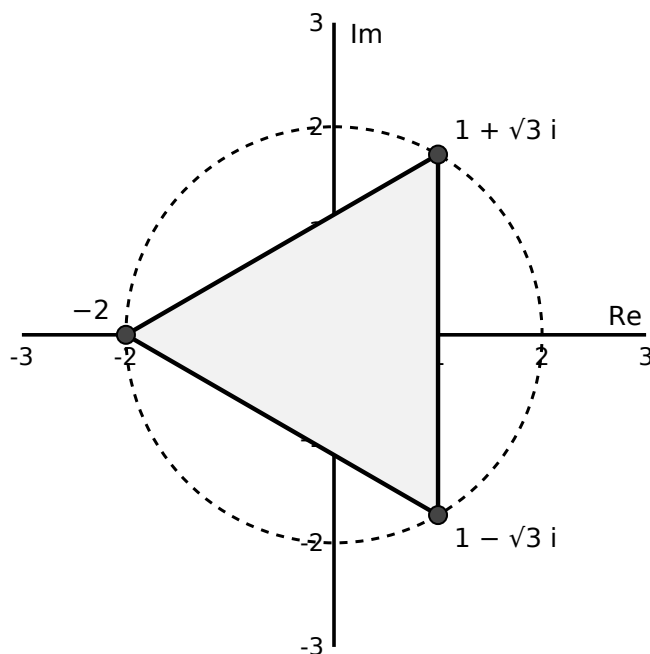
(a) $3^5 = 243$ and $5 \cdot 12^\circ = 60^\circ$, so $243(\cos 60^\circ + i \sin 60^\circ)$.

(b) $\sqrt{3} - i = 2(\cos(-30^\circ) + i \sin(-30^\circ))$. The seventh power is $2^7(\cos(-210^\circ) + i \sin(-210^\circ)) = 128(\cos 150^\circ + i \sin 150^\circ) = 128(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = -64\sqrt{3} + 64i$.

Answer: **(a) $243(\cos 60^\circ + i \sin 60^\circ)$; (b) $-64\sqrt{3} + 64i$**

Scoring: (a) 1 point. (b) 1 point for the polar form of $\sqrt{3} - i$ and 1 point for the correct power and final rectangular form.

Question 7 Cube Roots of -8 (3 points)



The three cube roots of -8 form an equilateral triangle on the circle of radius 2.

$-8 = 8(\cos 180^\circ + i \sin 180^\circ)$. The cube roots have modulus 2 and angles $(180^\circ + 360^\circ k)/3 = 60^\circ, 180^\circ, 300^\circ$.

$2(\cos 60^\circ + i \sin 60^\circ) = 1 + \sqrt{3}i$; $2(\cos 180^\circ + i \sin 180^\circ) = -2$; $2(\cos 300^\circ + i \sin 300^\circ) = 1 - \sqrt{3}i$.

Answer: **$1 + \sqrt{3}i, -2, 1 - \sqrt{3}i$**

Scoring: 1 point for the modulus and angles, 2 points for the three correct roots (1 point if only two).

Question 8 Scaling and Turning (2 points)

(a) $3i(5 + 2i) = 15i + 6i^2 = -6 + 15i$.

(b) $|P| = \sqrt{29}$ and $|Q| = \sqrt{(36 + 225)} = \sqrt{261} = 3\sqrt{29}$. Q is three times as far from the origin as P, and it is P turned 90° counterclockwise.

Answer: **$Q = -6 + 15i$; $|Q| = 3|P|$**

Scoring: 1 point for each part.

Question 9 Modulus of a Product (2 points)

$zw = 3 - 12i + i - 4i^2 = 7 - 11i$.

$|zw| = \sqrt{(49 + 121)} = \sqrt{170}$. Also $|z| = \sqrt{10}$ and $|w| = \sqrt{17}$, and $\sqrt{10} \cdot \sqrt{17} = \sqrt{170}$.

Answer: **$zw = 7 - 11i$ and both sides equal $\sqrt{170}$**

Scoring: 1 point for the product, 1 point for the comparison of moduli.