

Polar Form and Powers of Complex Numbers

Grade 12 – Precalculus math · Worksheet · Standards: N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3

Name: _____ Date: _____ Class: _____

These problems train the full toolkit for aligned standards N-CN.A.3, N-CN.B.4, N-CN.B.5 and N-CN.B.6: plotting, modulus, conjugates, converting between rectangular and polar form, and the product, quotient and power rules.

Problems climb from reading a point to roots of unity, alternating current and robot moves. Use exact values unless a decimal is requested.

Standards: N-CN.B.4, N-CN.B.5, N-CN.B.6, N-CN.A.3 · **Level:** Grade 12 Precalculus

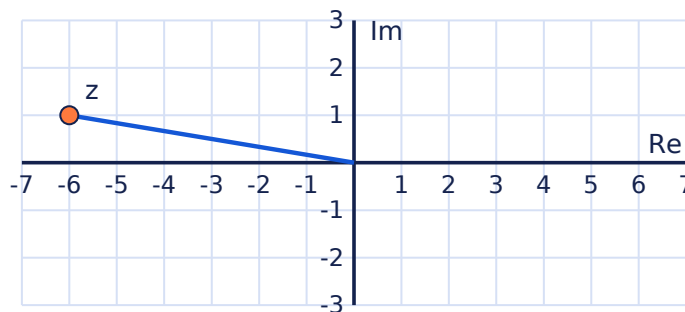
Problem 1 Modulus of a Complex Number (easy)

Find the modulus of $z = 2 - 5i$. Give an exact value and a decimal rounded to hundredths.

Problem 2 Conjugate and Product (easy)

Let $z = 3 - 7i$. Write the conjugate $\bar{z}$, then compute $z \cdot \bar{z}$ and compare it with $|z|^2$.

Problem 3 Reading a Point (easy)



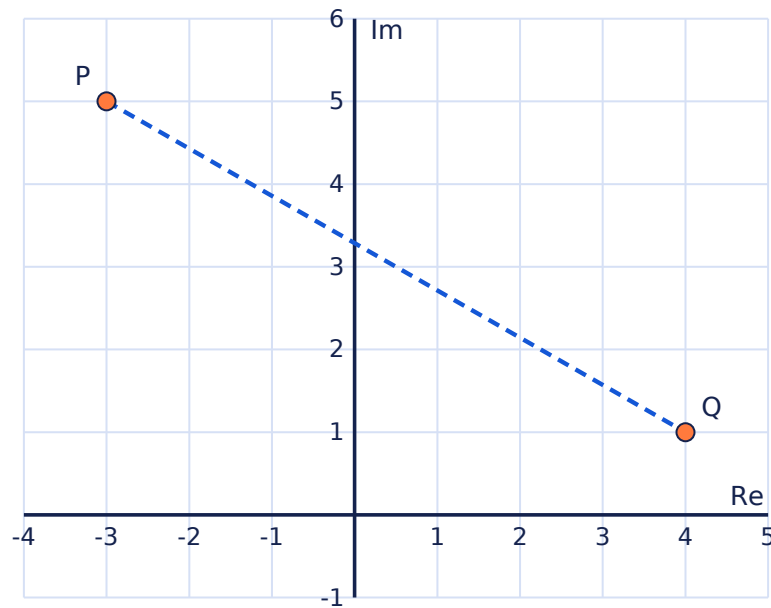
The point z in the complex plane.

Write the complex number z shown in the figure, state its conjugate, and find its modulus.

Problem 4 Modulus and Argument (easy)

Find the modulus and the argument (in degrees, $0^\circ \leq \theta < 360^\circ$) of $-5 + 5i$.

Problem 5 Distance and Midpoint (easy)



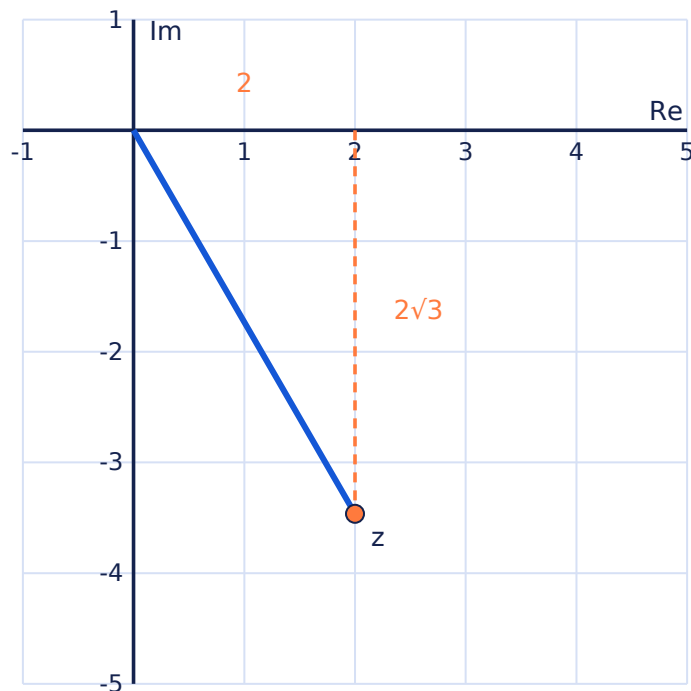
Points P and Q joined by a segment.

Points P and Q in the figure represent $-3 + 5i$ and $4 + i$. Find the distance PQ and the midpoint of the segment PQ.

Problem 6 Polar to Rectangular (easy)

Write $6(\cos 60^\circ + i \sin 60^\circ)$ in the form $a + bi$.

Problem 7 Rectangular to Polar (medium)



The point $z = 2 - 2\sqrt{3}i$ in Quadrant IV.

The figure shows $z = 2 - 2\sqrt{3}i$. Write z in polar form with $0^\circ \leq \theta < 360^\circ$.

Problem 8 Multiplying in Polar Form (medium)

Multiply $4(\cos 25^\circ + i \sin 25^\circ)$ by $5(\cos 35^\circ + i \sin 35^\circ)$. Give the result in polar form and in the form $a + bi$.

Problem 9 Dividing in Polar Form (medium)

Divide $18(\cos 200^\circ + i \sin 200^\circ)$ by $6(\cos 80^\circ + i \sin 80^\circ)$ and write the quotient in the form $a + bi$.

Problem 10 Product via Polar Form (medium)

Let $z_1 = 1 + i$ and $z_2 = 1 + \sqrt{3}i$. Write each in polar form, then give the modulus and an argument of $z_1 z_2$.

Problem 11 A Tenth Power (medium)

Write $1 - i$ in polar form first, then evaluate $(1 - i)^{10}$ with De Moivre's theorem.

Problem 12 Cubing in Polar Form (medium)

Evaluate $[2(\cos 20^\circ + i \sin 20^\circ)]^3$ and write the result in the form $a + bi$.

Problem 13 A Sixth Power (medium)

Evaluate $(\sqrt{3} + i)^6$.

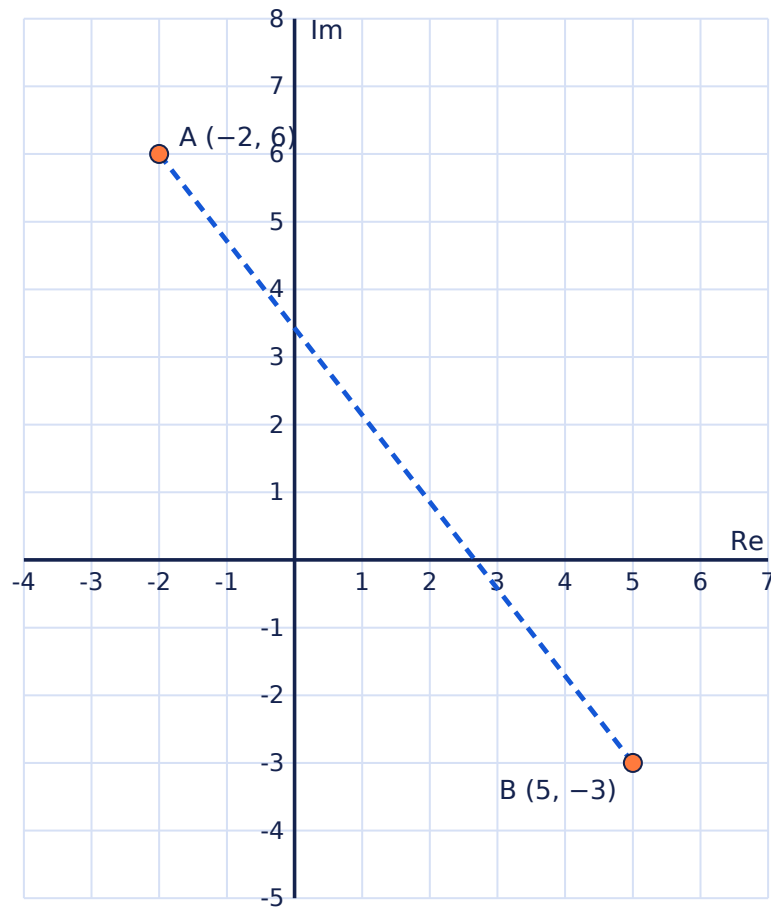
Problem 14 A Fifth Power (hard)

Evaluate $(-1 + i)^5$ and write the answer in the form $a + bi$.

Problem 15 Cube Roots of 64 (hard)

Find all three cube roots of 64 and write them in the form $a + bi$.

Problem 16 Trail Distance (hard)



Trail map: 1 unit = 1 mile. The dashed line is the straight-line distance.

On a park map, each unit of the complex plane is 1 mile. Trailhead A is at $-2 + 6i$ and trailhead B is at $5 - 3i$. A rest stop is built at the midpoint. Find the straight-line distance from A to B to the nearest hundredth of a mile and the location of the rest stop.

Problem 17 Robot Rotation (hard)

A warehouse robot starts at $3 + 2i$ (in meters). Each move multiplies its position by $2i$, which doubles its distance from the origin and turns it 90° counterclockwise. (a) Find its position after one move. (b) Find its position after four identical moves.

Problem 18 Alternating Current (hard)

In an AC circuit, a voltage of $48(\cos 70^\circ + i \sin 70^\circ)$ volts is applied to an impedance of $6(\cos 25^\circ + i \sin 25^\circ)$ ohms. The current is voltage $\div$ impedance. Find the current in polar form and in the form $a + bi$ (decimals to hundredths).

Problem 19 Sixth Roots of Unity (hard)

(a) Show that $z = \cos 60^\circ + i \sin 60^\circ$ satisfies $z^6 = 1$. (b) List all six sixth roots of 1 in the form $a + bi$. (c) Describe the polygon they form.

Problem 20 A Quotient of Powers (hard)

Simplify $(1 - \sqrt{3}i)^5 \div (1 + i)^4$ and write the result in the form $a + bi$.