

Parabolas, Ellipses and Hyperbolas in Standard Form

Grade 12 – Precalculus math · Test Answer Key · Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3

Use the scoring notes for partial credit. Most items award one point for the standard form or key value and one point for the final answer. Exact radicals are expected.

Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · **Level:** Grade 12 Precalculus

Question 1 Circle in General Form (2 points)

$(x^2 + 10x + 25) + (y^2 - 6y + 9) = -18 + 25 + 9$, so $(x + 5)^2 + (y - 3)^2 = 16$.

Answer: **center $(-5, 3)$, radius 4**

Scoring: 1 point for completing both squares, 1 point for the center and radius.

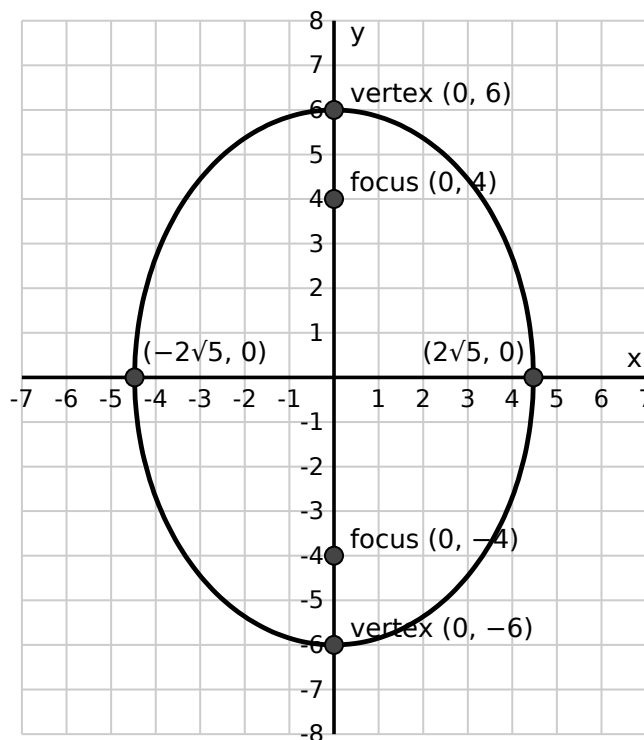
Question 2 A Downward Parabola (2 points)

The directrix is 3 units above the vertex, so the parabola opens downward with $p = -3$. The equation is $(x + 1)^2 = 4(-3)(y - 4) = -12(y - 4)$. The focus is $(-1, 4 - 3) = (-1, 1)$.

Answer: **$(x + 1)^2 = -12(y - 4)$; focus $(-1, 1)$**

Scoring: 1 point for the equation, 1 point for the focus.

Question 3 Reading an Ellipse (3 points)



An ellipse centered at the origin.

The larger denominator 36 is under y , so the major axis is vertical with $a = 6$ and $b = \sqrt{20} = 2\sqrt{5}$. Then $c^2 = 36 - 20 = 16$ and $c = 4$.

(a) Vertices $(0, \pm 6)$. (b) Minor axis endpoints $(\pm 2\sqrt{5}, 0)$. (c) Foci $(0, \pm 4)$.

Answer: **$(0, \pm 6)$; $(\pm 2\sqrt{5}, 0)$; $(0, \pm 4)$**

Scoring: 1 point for each part.

Question 4 A Hyperbola from Foci (3 points)

(a) $c = 7$ and $a = 4$, so $b^2 = c^2 - a^2 = 49 - 16 = 33$. The equation is $\frac{x^2}{16} - \frac{y^2}{33} = 1$.

(b) $y = \pm \frac{b}{a}x = \pm \frac{\sqrt{33}}{4}x$.

Answer: $\frac{x^2}{16} - \frac{y^2}{33} = 1$; asymptotes $y = \pm \frac{\sqrt{33}}{4}x$

Scoring: 1 point for b^2 , 1 point for the equation, 1 point for the asymptotes.

Question 5 Identify the Conic (2 points)

$4(x^2 - 6x) + (y^2 + 4y) = -24$. Complete the squares: $4(x - 3)^2 + (y + 2)^2 = -24 + 36 + 4 = 16$, so $\frac{(x - 3)^2}{4} + \frac{(y + 2)^2}{16} = 1$.

It is an ellipse with center $(3, -2)$, vertical major axis, $a = 4$, $b = 2$ and $c = \sqrt{12} = 2\sqrt{3}$. The foci

are $(3, -2 \pm 2\sqrt{3})$.

Answer: **ellipse; center $(3, -2)$; foci $(3, -2 \pm 2\sqrt{3})$**

Scoring: 1 point for the standard form and the type, 1 point for the center and foci.

Question 6 Parabolic Mirror (2 points)

The rim point is $(12, 4)$. In $x^2 = 4py$: $144 = 16p$, so $p = 9$. The equation is $x^2 = 36y$ and the focus is at $(0, 9)$, so the source sits 9 centimeters from the vertex.

Answer: **$x^2 = 36y$; 9 cm from the vertex**

Scoring: 1 point for the equation, 1 point for the distance.

Question 7 Running Track (3 points)

(a) $a = 60$ and $b = 40$: $\frac{x^2}{3600} + \frac{y^2}{1600} = 1$.

(b) $c^2 = 3600 - 1600 = 2000$, so $c = 20\sqrt{5} \approx 44.7$ m.

(c) The foci are $2c \approx 89.4$ m apart.

Answer: **$\frac{x^2}{3600} + \frac{y^2}{1600} = 1$; about 44.7 m; about 89.4 m**

Scoring: 1 point for each part.

Question 8 Checking the Definition (3 points)

(a) $c^2 = 49 - 40 = 9$, so the foci are $(\pm 3, 0)$.

(b) $\frac{0}{49} + \frac{(2\sqrt{10})^2}{40} = \frac{40}{40} = 1$.

(c) The distance from $(0, 2\sqrt{10})$ to $(3, 0)$ is $\sqrt{(9 + 40)} = 7$, and to $(-3, 0)$ it is also 7. The sum is $14 = 2(7) = 2a$.

Answer: **foci $(\pm 3, 0)$; each focal distance is 7, with sum 14**

Scoring: 1 point for each part.