

Parabolas, Ellipses and Hyperbolas in Standard Form

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3

Each answer names the type of conic, shows the values of a , b , c or p , and checks the result against the definition where possible. Watch which relationship, $a^2 - b^2$ or $a^2 + b^2$, applies.

Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · **Level:** Grade 12 Precalculus

Problem 1 Circle from Its Equation

Compare with $(x - h)^2 + (y - k)^2 = r^2$. Here $h = 6$ and $k = -1$, and $r^2 = 20$, so $r = \sqrt{20} = 2\sqrt{5} \approx 4.47$.

Answer: **center (6, -1), radius $2\sqrt{5}$**

Problem 2 Circle from a Point

$r^2 = (-1 - (-7))^2 + (-1 - 4)^2 = 36 + 25 = 61$.

Answer: **$(x + 7)^2 + (y - 4)^2 = 61$**

Problem 3 Focus and Directrix

Compare with $x^2 = 4py$: $4p = 12$, so $p = 3$. The vertex is $(0, 0)$, the focus is $(0, 3)$ and the directrix is $y = -3$.

Answer: **vertex (0, 0), focus (0, 3), directrix $y = -3$**

Problem 4 A Sideways Parabola

$4p = -20$ gives $p = -5$. The focus is $(-5, 0)$ and the directrix is $x = 5$. Since p is negative, the parabola opens to the left.

Answer: **focus (-5, 0), directrix $x = 5$, opens left**

Problem 5 Equation from the Figure

The focus is 4 units below the vertex, so $p = -4$. The equation is $x^2 = 4py = -16y$. The directrix

is $y = 4$.

Answer: $x^2 = -16y$

Problem 6 Parts of an Ellipse

$a^2 = 49$ (under x), so $a = 7$, and $b^2 = 25$, so $b = 5$. The major axis is horizontal.

$c^2 = 49 - 25 = 24$, so $c = 2\sqrt{6} \approx 4.90$.

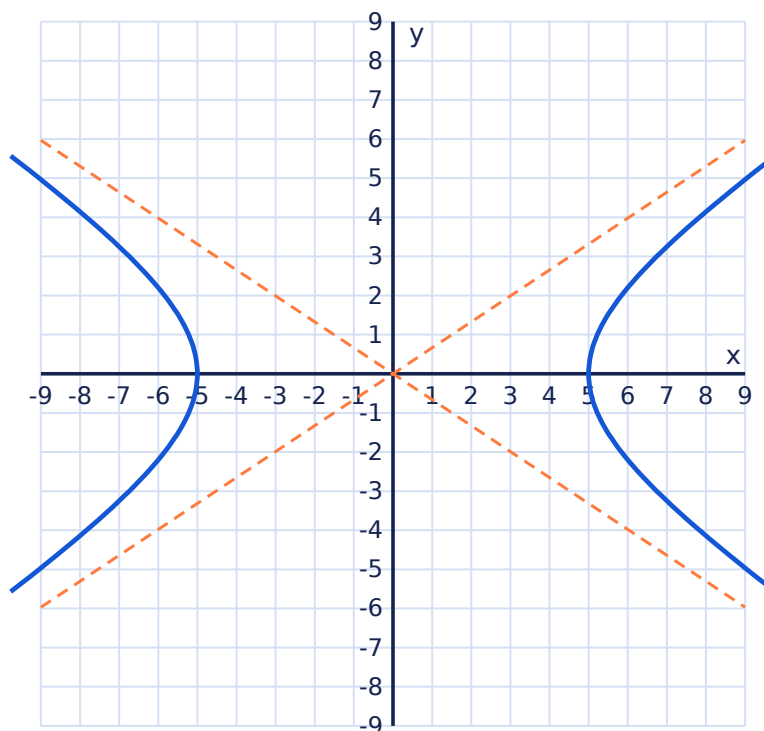
Answer: **vertices $(\pm 7, 0)$, minor axis endpoints $(0, \pm 5)$, foci $(\pm 2\sqrt{6}, 0)$**

Problem 7 Ellipse from Vertices and Foci

The major axis is vertical with $a = 9$. The focal distance is $c = 5$, so $b^2 = a^2 - c^2 = 81 - 25 = 56$.

Answer: $\frac{x^2}{56} + \frac{y^2}{81} = 1$

Problem 8 Parts of a Hyperbola



The hyperbola with dashed asymptotes.

$a = 5$ and $b = \sqrt{11}$. $c^2 = 25 + 11 = 36$, so $c = 6$. The main axis is horizontal.

The asymptotes are $y = \pm \frac{\sqrt{11}}{5}x$.

Answer: **vertices $(\pm 5, 0)$, foci $(\pm 6, 0)$, asymptotes $y = \pm \frac{\sqrt{11}}{5}x$**

Problem 9 A Vertical Hyperbola

$a^2 = 9$ so $a = 3$, and $b^2 = 27$ so $b = 3\sqrt{3}$. $c^2 = 9 + 27 = 36$, so $c = 6$. The main axis is vertical.

The asymptotes are $y = \pm \frac{a}{b}x = \pm \frac{3}{3\sqrt{3}}x = \pm \frac{\sqrt{3}}{3}x$.

Answer: **vertices (0, ± 3), foci (0, ± 6), asymptotes $y = \pm \frac{\sqrt{3}}{3}x$**

Problem 10 Shifted Parabola

The focus is 5 units above the vertex, so $p = 5$. The equation is $(x - 5)^2 = 4(5)(y + 2) = 20(y + 2)$. The directrix is $y = -2 - 5 = -7$.

Answer: **$(x - 5)^2 = 20(y + 2)$, directrix $y = -7$**

Problem 11 Completing the Square: Circle

Group: $(x^2 - 8x) + (y^2 + 10y) = -5$. Add 16 and 25 to both sides: $(x - 4)^2 + (y + 5)^2 = -5 + 16 + 25 = 36$.

Answer: **$(x - 4)^2 + (y + 5)^2 = 36$, center (4, -5), radius 6**

Problem 12 Completing the Square: Parabola

Move the x term and constant: $y^2 - 4y = 12x - 40$. Add 4: $(y - 2)^2 = 12x - 36 = 12(x - 3)$.

So $4p = 12$ and $p = 3$. The vertex is (3, 2), the focus is (6, 2) and the directrix is $x = 0$.

Answer: **$(y - 2)^2 = 12(x - 3)$, vertex (3, 2), focus (6, 2), directrix $x = 0$**

Problem 13 Completing the Square: Ellipse

$9(x^2 - 2x) + 4(y^2 + 4y) = 11$. Complete the squares: $9(x - 1)^2 + 4(y + 2)^2 = 11 + 9 + 16 = 36$.

Divide by 36: $\frac{(x - 1)^2}{4} + \frac{(y + 2)^2}{9} = 1$. It is an ellipse with center (1, -2), vertical major axis, $a = 3$, $b = 2$ and $c = \sqrt{5}$.

Answer: **ellipse; center (1, -2); vertices (1, 1) and (1, -5); foci (1, $-2 \pm \sqrt{5}$)**

Problem 14 Completing the Square: Hyperbola

$4(x^2 + 6x) - 5(y^2 - 2y) = -11$. Complete the squares, remembering the factors: $4(x + 3)^2 - 5(y - 1)^2 = -11 + 36 - 5 = 20$.

Divide by 20: $\frac{(x+3)^2}{5} - \frac{(y-1)^2}{4} = 1$. So $a^2 = 5$, $b^2 = 4$ and $c^2 = 9$, $c = 3$. The center is $(-3, 1)$ with a horizontal main axis.

Foci: $(-3 \pm 3, 1) = (0, 1)$ and $(-6, 1)$. Asymptotes: $y - 1 = \pm \frac{2}{\sqrt{5}}(x + 3)$.

Answer: **center $(-3, 1)$, foci $(0, 1)$ and $(-6, 1)$, asymptotes $y - 1 = \pm \frac{2}{\sqrt{5}}(x + 3)$**

Problem 15 Satellite Dish

(a) Use $x^2 = 4py$. The rim point is $(5, 1.25)$, so $25 = 4p(1.25) = 5p$ and $p = 5$. The equation is $x^2 = 20y$.

(b) The focus is at $(0, p) = (0, 5)$, so the receiver sits 5 feet above the vertex.

Answer: **(a) $x^2 = 20y$; (b) 5 feet above the vertex**

Problem 16 Whispering Hall

(a) $a = 40$ and $b = 15$, so $\frac{x^2}{1600} + \frac{y^2}{225} = 1$.

(b) $c^2 = 1600 - 225 = 1375$, so $c = \sqrt{1375} \approx 37.1$ feet.

(c) The foci are $2c \approx 74.2$ feet apart.

Answer: **(a) $\frac{x^2}{1600} + \frac{y^2}{225} = 1$; (b) about 37.1 ft; (c) about 74.2 ft**

Problem 17 Elliptical Bridge Arch

(a) $a = 30$ and $b = 18$: $\frac{x^2}{900} + \frac{y^2}{324} = 1$ with $y \geq 0$.

(b) At $x = 20$: $y = 18\sqrt{1 - \frac{400}{900}} = 18\sqrt{\frac{5}{9}} = 6\sqrt{5} \approx 13.4$ feet.

(c) At $y = 14$: $x = 30\sqrt{1 - \frac{196}{324}} = 30\sqrt{\frac{128}{324}} \approx 18.9$. The truck can be at most $2(18.9) \approx 37.7$ feet wide.

Answer: **(a) $\frac{x^2}{900} + \frac{y^2}{324} = 1$; (b) about 13.4 ft; (c) about 37.7 ft**

Problem 18 Radio Station Hyperbola

(a) The foci are at $(\pm 90, 0)$, so $c = 90$. The difference of distances is $2a = 100$, so $a = 50$. Then $b^2 = c^2 - a^2 = 8100 - 2500 = 5600$. The equation is $\frac{x^2}{2500} - \frac{y^2}{5600} = 1$.

(b) At the vertices $(\pm 50, 0)$.

(c) $y = \pm \sqrt{\frac{5600}{50}}x = \pm \frac{20\sqrt{14}}{50}x = \pm \frac{2\sqrt{14}}{5}x \approx \pm 1.50x$.

Answer: **(a)** $\frac{x^2}{2500} - \frac{y^2}{5600} = 1$; **(b)** $(\pm 50, 0)$; **(c)** $y \approx \pm 1.50x$

Problem 19 A Comet's Orbit

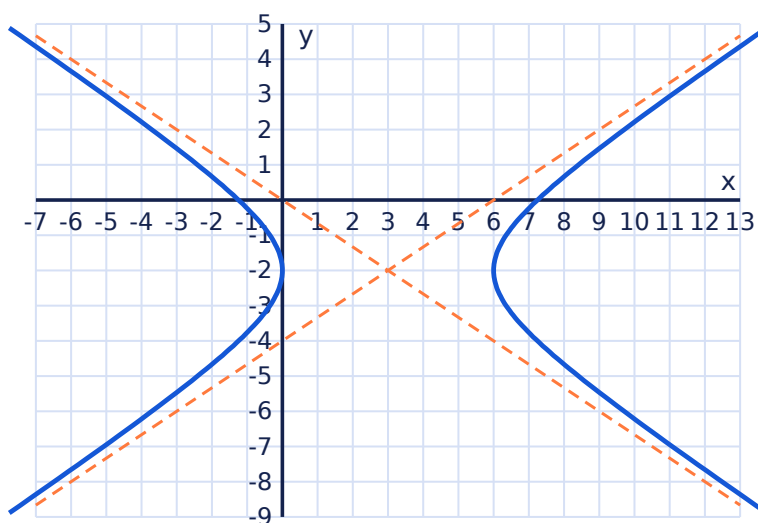
(a) The major axis is $2a = 52$, so $a = 26$. The closest distance is $a - c = 10$, so $c = 16$.

(b) $b^2 = a^2 - c^2 = 676 - 256 = 420$. The equation is $\frac{x^2}{676} + \frac{y^2}{420} = 1$.

(c) The greatest distance is $a + c = 42$ million miles.

Answer: **(a)** $a = 26$, $c = 16$; **(b)** $\frac{x^2}{676} + \frac{y^2}{420} = 1$; **(c)** 42 million miles

Problem 20 Identify and Describe



The hyperbola with dashed asymptotes.

Group: $4(x^2 - 6x) - 9(y^2 + 4y) = 36$. Complete the squares: $4(x - 3)^2 - 9(y + 2)^2 = 36 + 36 - 36 = 36$.

Divide by 36: $\frac{(x - 3)^2}{9} - \frac{(y + 2)^2}{4} = 1$. The minus sign makes it a hyperbola with center $(3, -2)$, $a = 3$, $b = 2$ and $c = \sqrt{13}$.

Vertices: $(0, -2)$ and $(6, -2)$. Foci: $(3 \pm \sqrt{13}, -2)$. Asymptotes: $y + 2 = \pm \frac{2}{3}(x - 3)$.

Answer: $\frac{(x - 3)^2}{9} - \frac{(y + 2)^2}{4} = 1$; **center $(3, -2)$; vertices $(0, -2)$, $(6, -2)$; foci $(3 \pm \sqrt{13}, -2)$**