

Parabolas, Ellipses and Hyperbolas in Standard Form

Grade 12 – Precalculus math · Worksheet · Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3

Name: _____ Date: _____ Class: _____

These problems cover circles, parabolas, ellipses and hyperbolas from their definitions with foci and directrices (G-GPE.A.1, G-GPE.A.2 and G-GPE.A.3). Students find standard forms, vertices, foci and asymptotes, and complete the square.

The last problems model dishes, halls, bridges, navigation and orbits. Give exact values and decimals where asked.

Standards: G-GPE.A.1, G-GPE.A.2, G-GPE.A.3 · **Level:** Grade 12 Precalculus

Problem 1 Circle from Its Equation (easy)

Find the center and radius of $(x - 6)^2 + (y + 1)^2 = 20$.

Problem 2 Circle from a Point (easy)

A circle is centered at $(-7, 4)$ and its edge goes through $(-1, -1)$. Give its equation in standard form.

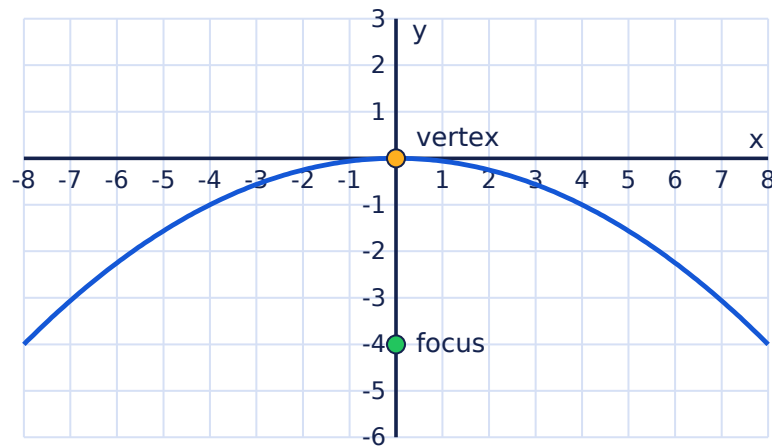
Problem 3 Focus and Directrix (easy)

For the parabola $x^2 = 12y$, find the vertex, focus and directrix.

Problem 4 A Sideways Parabola (easy)

For $y^2 = -20x$, find the focus and directrix and state which way the parabola opens.

Problem 5 Equation from the Figure (easy)



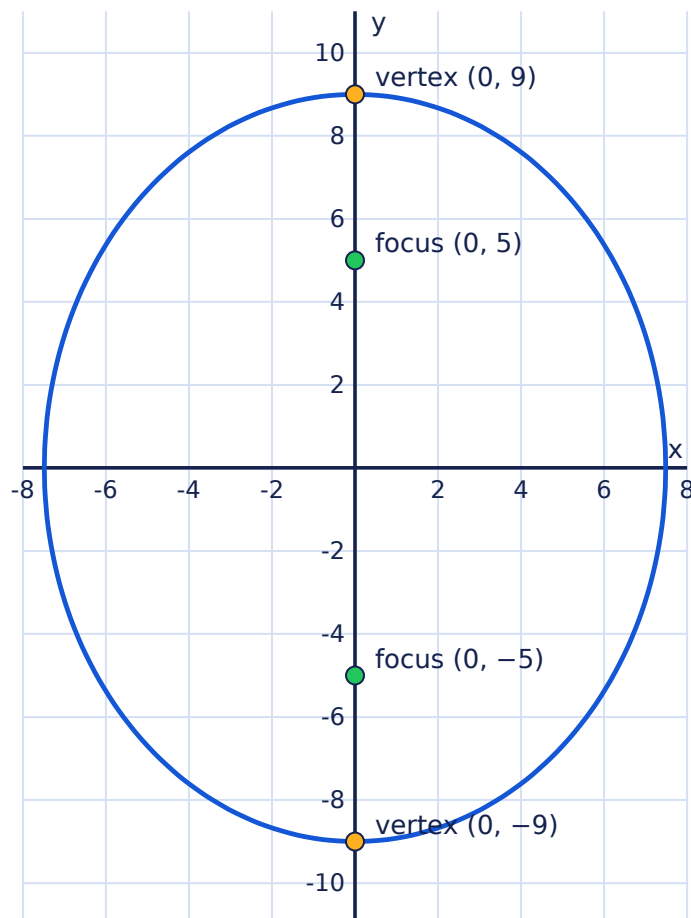
A parabola with its focus marked.

The parabola in the figure has its vertex at the origin and its focus at $(0, -4)$. Write its equation.

Problem 6 Parts of an Ellipse (easy)

For $\frac{x^2}{49} + \frac{y^2}{25} = 1$, find the vertices, the endpoints of the minor axis and the foci.

Problem 7 Ellipse from Vertices and Foci (medium)



An ellipse with labeled vertices and foci.

An ellipse centered at the origin has vertices $(0, \pm 9)$ and foci $(0, \pm 5)$, as shown. Write its equation.

Problem 8 Parts of a Hyperbola (medium)

For $\frac{x^2}{25} - \frac{y^2}{11} = 1$, find the vertices, the foci and the equations of the asymptotes.

Problem 9 A Vertical Hyperbola (medium)

For $\frac{y^2}{9} - \frac{x^2}{27} = 1$, find the vertices, the foci and the asymptotes.

Problem 10 Shifted Parabola (medium)

A parabola has vertex $(5, -2)$ and focus $(5, 3)$. Write its equation and find its directrix.

Problem 11 Completing the Square: Circle (medium)

Rewrite $x^2 + y^2 - 8x + 10y + 5 = 0$ in standard form and give the center and radius.

Problem 12 Completing the Square: Parabola (medium)

Rewrite $y^2 - 4y - 12x + 40 = 0$ in standard form. Find the vertex, focus and directrix.

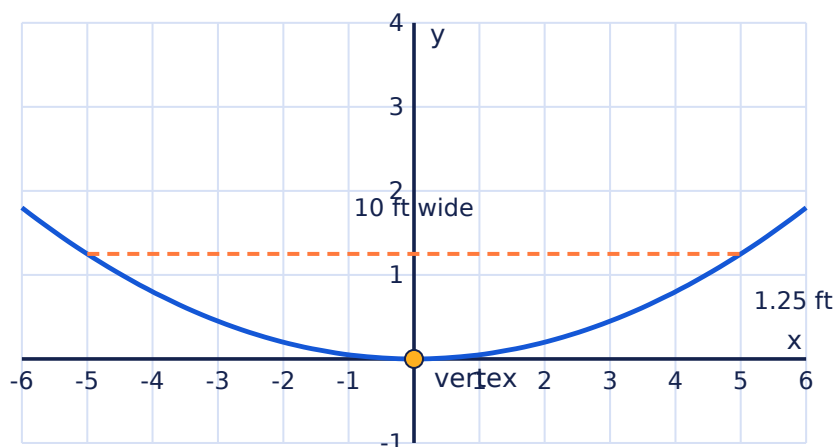
Problem 13 Completing the Square: Ellipse (medium)

Identify $9x^2 + 4y^2 - 18x + 16y - 11 = 0$ and find its center, vertices and foci.

Problem 14 Completing the Square: Hyperbola (hard)

Rewrite $4x^2 - 5y^2 + 24x + 10y + 11 = 0$ in standard form. Find the center, foci and asymptotes.

Problem 15 Satellite Dish (hard)

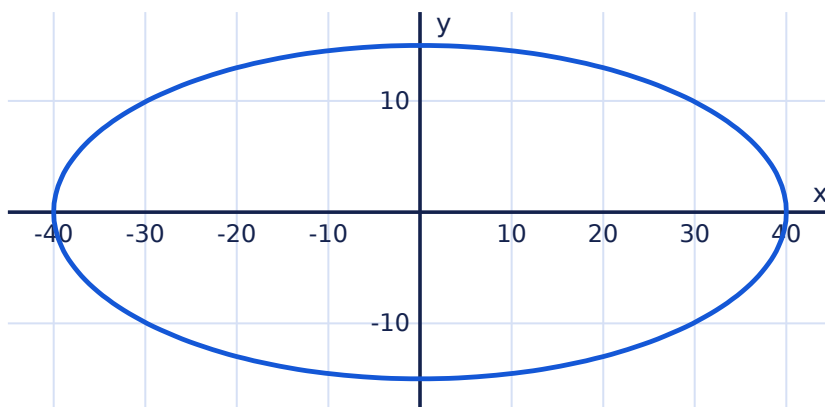


Cross-section of the dish.

A satellite dish is 10 feet wide and 1.25 feet deep. Model its cross-section as a parabola with its

vertex at the origin opening upward. (a) Write the equation. (b) How far above the vertex should the receiver be placed, at the focus?

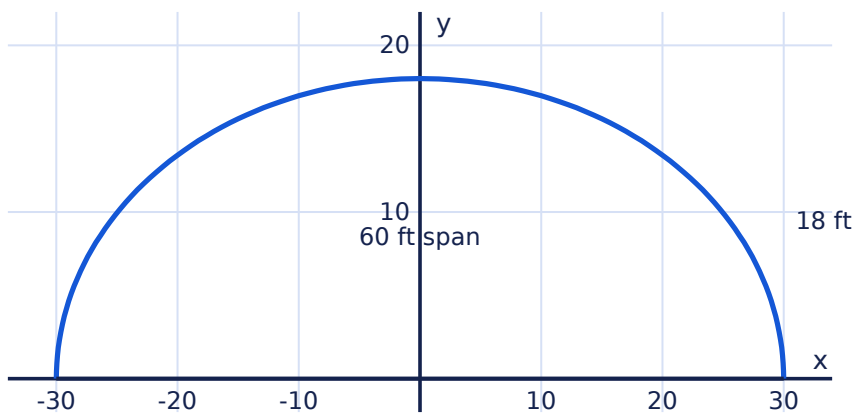
Problem 16 Whispering Hall (hard)



Top view of the whispering hall.

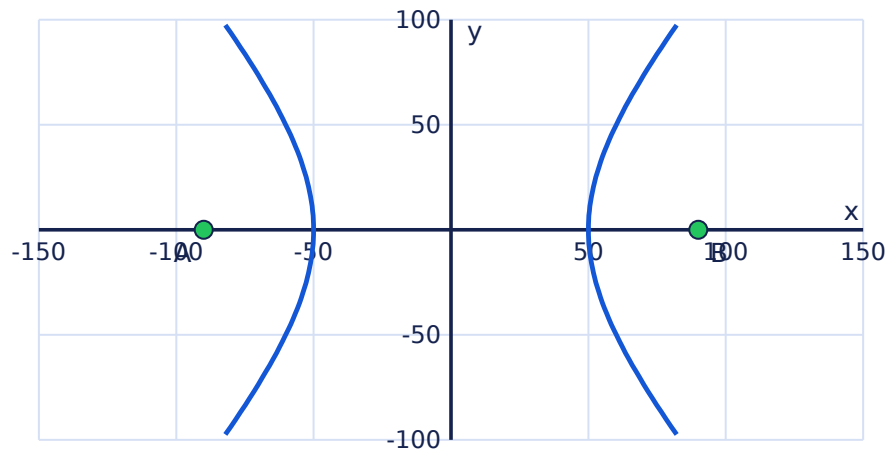
A hall is shaped like an ellipse 80 feet long and 30 feet wide. Two people stand at the foci and whisper to each other. (a) Write the equation of the ellipse with its center at the origin. (b) How far from the center is each focus, to the nearest tenth of a foot? (c) How far apart are the two people?

Problem 17 Elliptical Bridge Arch (hard)



A semi-elliptical arch.

A bridge arch is the upper half of an ellipse 60 feet wide at the base and 18 feet high at the center. (a) Write an equation with the center of the base at the origin. (b) Find the height of the arch 20 feet from the center. (c) A truck 14 feet tall drives under the arch. How wide can the truck be if it is centered? Round to the nearest tenth.

Problem 18 Radio Station Hyperbola (hard)

Station positions and a hyperbolic line of position.

Two radio stations are 180 miles apart. A ship measures that it is 100 miles closer to one station than the other. Place the stations at $(\pm 90, 0)$. (a) Write the equation of the hyperbola of possible ship positions. (b) Where does it cross the x-axis? (c) Find its asymptotes.

Problem 19 A Comet's Orbit (hard)

A comet travels on an ellipse with the sun at one focus. The major axis is 52 million miles long, and at its closest the comet is 10 million miles from the sun. (a) Find a and c . (b) Find b^2 and write an equation with the center at the origin. (c) Find the greatest distance from the sun.

Problem 20 Identify and Describe (hard)

Show that $4x^2 - 9y^2 - 24x - 36y - 36 = 0$ is a hyperbola. Give its standard form, center, vertices, foci and asymptotes.