

# Counting Methods and Probability Rules

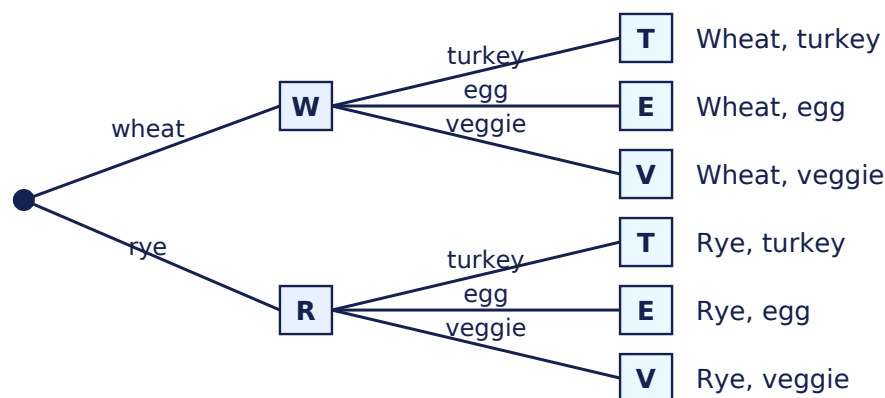
Grade 12 – Precalculus math · Lesson · Standards: S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2

**Quick answer:** The fundamental counting principle multiplies the number of ways to make each choice. Permutations count ordered selections and combinations count unordered groups. Probabilities of several events multiply with the general multiplication rule, and the expected value of a random variable is the probability-weighted average of its values.

**Standards:** S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2 · **Level:** Grade 12 Precalculus

## How does the fundamental counting principle work?

When a first decision has  $m$  options and a second has  $n$  options, the pair of decisions has  $m \times n$  outcomes, and the same idea chains through any number of stages. A deli with 2 breads and 3 fillings serves  $2 \times 3 = 6$  distinct sandwiches.



$$2 \text{ breads} \times 3 \text{ fillings} = 6 \text{ sandwiches.}$$

## When do you use permutations and combinations?

| Situation                       | Formula                         | Example                                           |
|---------------------------------|---------------------------------|---------------------------------------------------|
| Order matters (a ranked list)   | $P(n, r) = \frac{n!}{(n-r)!}$   | $P(8, 3) = 8 \cdot 7 \cdot 6 = 336$ podium orders |
| Order does not matter (a group) | $C(n, r) = \frac{n!}{r!(n-r)!}$ | $C(8, 3) = 56$ three-person teams                 |

### Common mistake

Using a permutation for a group. Ask yourself: would swapping two chosen items make a different outcome? If not, use a combination.

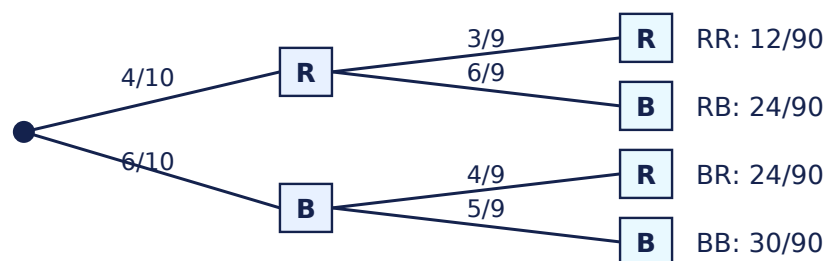
## How do you find a probability by counting?

When every outcome is equally likely, divide the outcomes you want by all outcomes. Suppose 3 people are picked from 5 juniors and 4 seniors. The committees with exactly 2 seniors number  $C(4, 2) \cdot C(5, 1) = 6 \cdot 5 = 30$ , while all committees number  $C(9, 3) = 84$ , so the chance is  $\frac{30}{84} = \frac{5}{14}$ .

## How does the general multiplication rule work?

### General multiplication rule

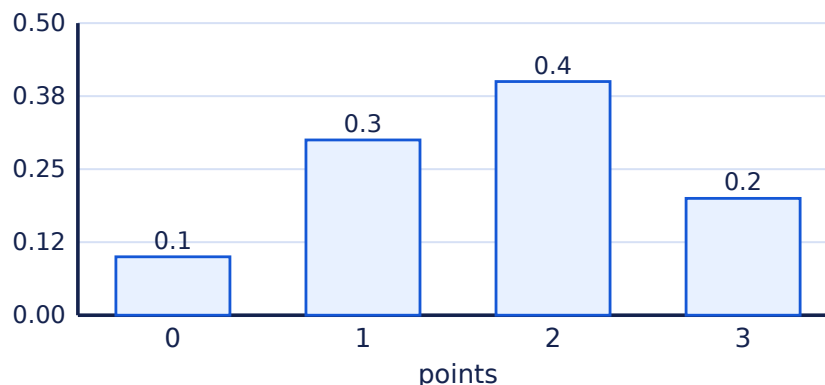
$P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$ . The second factor is the chance of B once A is known to have occurred. When A does not affect B, it is simply  $P(B)$ .



Pulling two red marbles in a row from this bag has probability  $\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$ , the red-red branch of the tree.

## What is expected value?

A random variable  $X$  attaches a number to every outcome. Its mean, the expected value, weights each number by its probability:  $E(X) = \sum x \cdot P(x)$ . For the dart distribution below,  $E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 1.7$  points per throw.



Probability distribution of the random variable.

### Mathbot's tip

Before computing, decide in one sentence whether order matters and whether items repeat. That sentence picks the formula.

### Key points to remember

- Counting principle: multiply the number of options at each stage.
- $P(n, r) = n!/(n - r)!$  counts ordered picks;  $C(n, r) = n!/(r!(n - r)!)$  counts groups.
- Equally likely outcomes:  $P(\text{event}) = \text{favorable} \div \text{total}$ .
- General multiplication rule:  $P(A \text{ and } B) = P(A) \cdot P(B | A)$ .
- Expected value:  $E(X) = \sum x \cdot P(x)$ .

## Counting Methods and Probability Rules: frequently asked questions

### What is the difference between a permutation and a combination?

A permutation counts selections where order matters, such as a ranked podium. A combination counts selections where order does not matter, such as a team. For the same  $n$  and  $r$ ,  $P$  is  $r!$  times larger than  $C$ .

### What does $P(B | A)$ mean?

It is the probability of  $B$  given that  $A$  has already happened. Drawing without replacement changes what remains, so  $P(B | A)$  is usually different from  $P(B)$ .

### How do I count "at least one" outcomes?

Subtract the complement from 1.  $P(\text{at least one}) = 1 - P(\text{none})$ . This is much shorter than adding the probabilities of one, two, three or more.

### Does expected value have to be a possible outcome?

No. It is a long-run average. A game that pays 0, 1, 2 or 3 points can have an expected value of 1.7 even though 1.7 is never scored in a single play.

### When do probabilities multiply and when do they add?

Multiply for "and" events that happen in sequence, using the conditional probability when the events depend on each other. Add for "or" events that cannot happen together.