

Counting Methods and Probability Rules

Grade 12 – Precalculus math · Test Answer Key · Standards: S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2

Use the scoring notes for partial credit. Most items award a point for the correct setup, such as choosing a permutation or a combination, and a point for the final value.

Standards: S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2 · **Level:** Grade 12 Precalculus

Question 1 Full Meals (2 points)

(a) $4 \times 6 \times 3 = 72$.

(b) Skipping dessert adds one more dessert option: $4 \times 6 \times (3 + 1) = 96$.

Answer: **(a) 72; (b) 96**

Scoring: 1 point for each part.

Question 2 Permutation or Combination (2 points)

$P(7, 3) = 7 \cdot 6 \cdot 5 = 210$. $C(7, 3) = \frac{210}{3!} = \frac{210}{6} = 35$.

Permutations count each group in every possible order, so they are $3! = 6$ times larger than combinations.

Answer: **210 and 35**

Scoring: 1 point for each value; the explanation is part of the second point.

Question 3 Random Committee (3 points)

(a) $C(10, 3) = 120$.

(b) $C(4, 2) \cdot C(6, 1) = 6 \cdot 6 = 36$ committees, so $P = \frac{36}{120} = \frac{3}{10}$.

(c) Use the complement. No girls: $C(6, 3) = 20$. $P(\text{at least one girl}) = 1 - \frac{20}{120} = \frac{5}{6}$.

Answer: **(a) 120; (b) $\frac{3}{10}$; (c) $\frac{5}{6}$**

Scoring: 1 point for each part.

Question 4 Trophy Shelf (2 points)

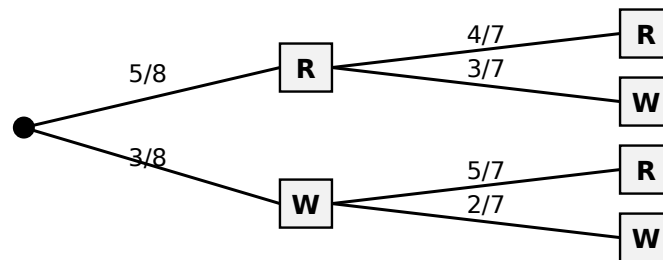
(a) $5! = 120$.

(b) The tallest trophy is fixed in the middle, and the other 4 can be arranged in $4! = 24$ ways.

Answer: **(a) 120; (b) 24**

Scoring: 1 point for each part.

Question 5 Marbles Without Replacement (3 points)



The completed tree.

(a) After a red draw, 4 red and 3 white remain among 7: $4/7$ and $3/7$. After a white draw, 5 red and 2 white remain among 7: $5/7$ and $2/7$.

(b) $\frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$.

(c) $\frac{5}{8} \cdot \frac{3}{7} + \frac{3}{8} \cdot \frac{5}{7} = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$.

Answer: **(b) $\frac{5}{14}$; (c) $\frac{15}{28}$**

Scoring: 1 point for each part.

Question 6 Expected Value of a Game (3 points)

(a) $0.4 + 0.3 + 0.2 + 0.1 = 1$, and every probability is between 0 and 1.

(b) $E(X) = (-2)(0.4) + 0(0.3) + 3(0.2) + 8(0.1) = -0.8 + 0 + 0.6 + 0.8 = 0.6$.

(c) Over many plays the player gains about \$0.60 per play on average.

Answer: **$E(X) = \$0.60$ per play**

Scoring: 1 point for each part.

Question 7 Die Game (3 points)

Winnings: \$12 with probability $\frac{1}{6}$, \$3 with probability $\frac{2}{6}$, \$0 with probability $\frac{3}{6}$.

$$E(\text{winnings}) = 12 \cdot \frac{1}{6} + 3 \cdot \frac{2}{6} + 0 = 2 + 1 = \$3.$$

Expected net gain = $3 - 5 = -\$2$. The player should expect to lose \$2 per game.

Answer: **expected winnings \$3; expected net -\$2; no**

Scoring: 1 point for the distribution, 1 point for the expected winnings, 1 point for the net gain and conclusion.

Question 8 Pets Survey (2 points)

Divide each count by 40: $P(0) = 0.20$, $P(1) = 0.40$, $P(2) = 0.25$, $P(3) = 0.15$.

$$E = 0(0.20) + 1(0.40) + 2(0.25) + 3(0.15) = 0.40 + 0.50 + 0.45 = 1.35.$$

Answer: **expected number of pets 1.35**

Scoring: 1 point for the distribution, 1 point for the expected value.