

Counting Methods and Probability Rules

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2

Each answer states whether order matters, shows the counting expression, and gives the final probability in lowest terms where possible. Read the checks to compare counting with the multiplication rule.

Standards: S-CP.B.8, S-CP.B.9, S-MD.A.1, S-MD.A.2 · **Level:** Grade 12 Precalculus

Problem 1 Sandwich Shop Combinations

Multiply the choices: $5 \times 4 \times 3 = 60$.

Answer: **60 sandwiches**

Problem 2 License Plates

26 choices for each letter and 10 for each digit: $26 \times 26 \times 10 \times 10 \times 10 = 676,000$.

Answer: **676,000 plates**

Problem 3 Ordered Picks

Order matters: $P(9, 3) = 9 \cdot 8 \cdot 7 = 504$.

Answer: **504 ways**

Problem 4 Choosing a Group

$C(10, 4) = \frac{(10 \cdot 9 \cdot 8 \cdot 7)}{(4 \cdot 3 \cdot 2 \cdot 1)} = \frac{5,040}{24} = 210$.

Answer: **210 groups**

Problem 5 Order or Not?

(a) The two offices are different, so order matters: $P(12, 2) = 12 \cdot 11 = 132$.

(b) Two delegates form an unordered pair: $C(12, 2) = \frac{(12 \cdot 11)}{2} = 66$.

Answer: **(a) 132 by permutation; (b) 66 by combination**

Problem 6 Candy Jar Probability

The jar holds $7 + 9 + 4 = 20$ candies.

(a) $P(\text{cherry}) = \frac{9}{20}$.

(b) Not lemon means $9 + 4 = 13$ candies, so $\frac{13}{20}$.

Answer: **(a) $\frac{9}{20}$; (b) $\frac{13}{20}$**

Problem 7 Books on a Shelf

(a) $6! = 720$.

(b) Glue the two books into one block: 5 items can be arranged in $5! = 120$ ways, and the two books can swap inside the block in 2 ways. So $120 \times 2 = 240$.

Answer: **(a) 720; (b) 240**

Problem 8 Starting Lineups

$$C(12, 5) = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5!} = \frac{95,040}{120} = 792.$$

Answer: **792 ways**

Problem 9 Arranging Letters

BANANAS has 7 letters: B once, A three times, N twice, S once.

Divide out the repeats: $\frac{7!}{(3! \cdot 2!)} = \frac{5,040}{12} = 420$.

Answer: **420 arrangements**

Problem 10 All Juniors

Favorable: $C(5, 4) = 5$. Total: $C(12, 4) = 495$.

$$P = \frac{5}{495} = \frac{1}{99} \approx 0.0101.$$

Answer: $\frac{1}{99}$

Problem 11 Two Blue Without Replacement

$P(\text{first blue}) = \frac{7}{12}$. After removing one blue, 6 blue remain among 11, so $P(\text{second blue} \mid \text{first$

blue) = $\frac{6}{11}$.

$$P(\text{both blue}) = \frac{7}{12} \cdot \frac{6}{11} = \frac{42}{132} = \frac{7}{22}.$$

Answer: $\frac{7}{22}$

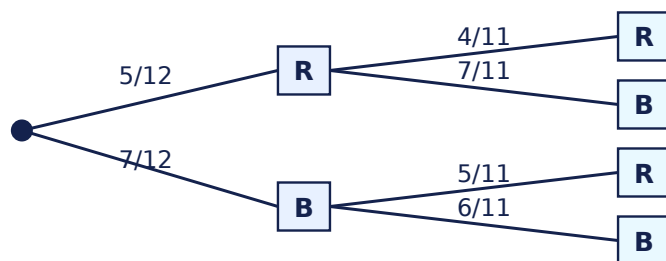
Problem 12 Red Then Blue

$P(\text{red first}) = \frac{5}{12}$. Then 7 blue remain among 11, so $P(\text{blue second} \mid \text{red first}) = \frac{7}{11}$.

$$P = \frac{5}{12} \cdot \frac{7}{11} = \frac{35}{132} \approx 0.265.$$

Answer: $\frac{35}{132}$

Problem 13 With or Without Replacement



Without replacement the second-draw probabilities change.

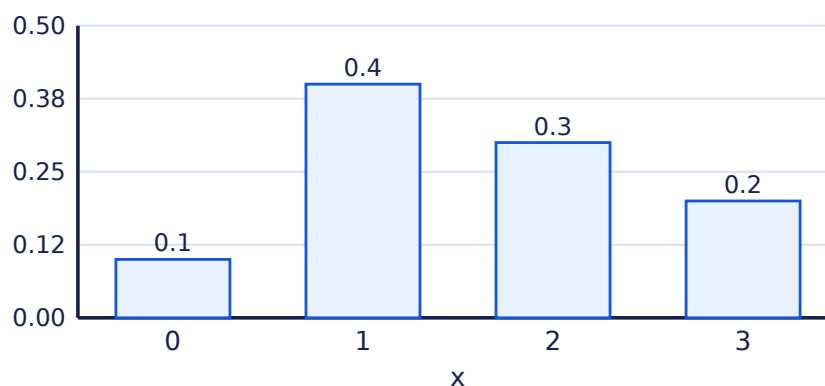
With replacement the draws are independent: $\frac{7}{12} \cdot \frac{7}{12} = \frac{49}{144} \approx 0.340$.

Without replacement the answer is $\frac{7}{22} \approx 0.318$.

Without replacement the probability is smaller, because removing a blue ball leaves fewer blue balls.

Answer: $\frac{49}{144} \approx \mathbf{0.340}$ compared with $\frac{7}{22} \approx \mathbf{0.318}$

Problem 14 Expected Value from a Table



Probability distribution of the random variable.

The probabilities add to $0.1 + 0.4 + 0.3 + 0.2 = 1$. $E(X) = 0(0.1) + 1(0.4) + 2(0.3) + 3(0.2) = 0 + 0.4 + 0.6 + 0.6 = 1.6$.

Answer: **1.6**

Problem 15 Carnival Spinner

Expected winnings: $10(0.1) + 3(0.3) + 0(0.6) = 1 + 0.9 = \$1.90$.

Net gain = $1.90 - 2.00 = -\$0.10$.

On average the player loses 10 cents per spin.

Answer: **\$1.90 expected winnings; expected net -\$0.10**

Problem 16 Password Count

The letters are chosen without repeats: $P(26, 4) = 26 \cdot 25 \cdot 24 \cdot 23 = 358,800$. The digits give $10 \times 10 = 100$.

Total = $358,800 \times 100 = 35,880,000$.

Answer: **35,880,000 passwords**

Problem 17 Committee with Boys

Use the complement. Total committees: $C(25, 5) = 53,130$.

No boys: $C(14, 5) = 2,002$. Exactly one boy: $C(11, 1) \cdot C(14, 4) = 11 \times 1,001 = 11,011$.

$P(\text{at most 1 boy}) = \frac{(2,002 + 11,011)}{53,130} = \frac{13,013}{53,130}$.

$P(\text{at least 2 boys}) = 1 - \frac{13,013}{53,130} = \frac{40,117}{53,130} \approx 0.755$.

Answer: **about 0.755**

Problem 18 Light Bulbs

$$P(\text{all work}) = \frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10} = \frac{336}{1,320} = \frac{14}{55} \approx 0.255.$$

$$\text{Check with combinations: } \frac{{}^{C(8,3)}}{{}^{C(12,3)}} = \frac{56}{220} = \frac{14}{55}.$$

Answer: $\frac{14}{55}$

Problem 19 Raffle Expected Value

$$\text{Total prize money: } 400 + 3(100) + 10(20) = 400 + 300 + 200 = \$900.$$

$$\text{Expected winnings per ticket} = \frac{900}{500} = \$1.80. \text{ Expected net gain} = 1.80 - 4.00 = -\$2.20.$$

Answer: **\$1.80 expected winnings; expected net -\$2.20**

Problem 20 Defective Phones

$$\text{Total pairs: } C(12, 2) = 66.$$

$$(a) P(0) = \frac{{}^{C(9,2)}}{66} = \frac{36}{66} = \frac{6}{11}. P(1) = \frac{{}^{(3 \cdot 9)}}{66} = \frac{27}{66} = \frac{9}{22}. P(2) = \frac{{}^{C(3,2)}}{66} = \frac{3}{66} = \frac{1}{22}. \text{ The three add to } 1.$$

$$(b) E(X) = 0 \cdot \frac{6}{11} + 1 \cdot \frac{9}{22} + 2 \cdot \frac{1}{22} = \frac{11}{22} = \frac{1}{2}.$$

Answer: **$P(0) = \frac{6}{11}$, $P(1) = \frac{9}{22}$, $P(2) = \frac{1}{22}$; $E(X) = \frac{1}{2}$**