

Expected Value and Smart Decisions

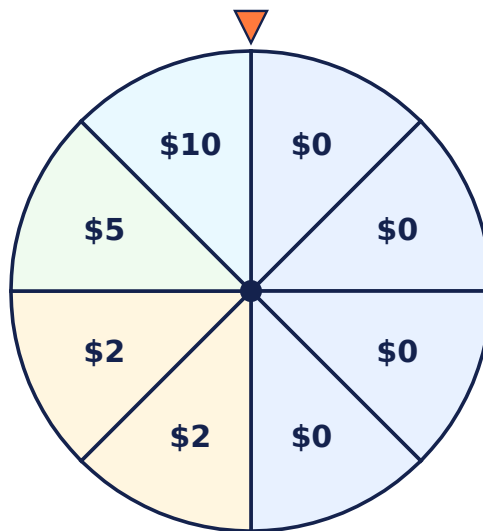
Grade 12 – Precalculus math · Lesson · Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5

Quick answer: A probability distribution lists every value of a random variable with its probability, and the probabilities must add to 1. The expected value is the sum of each value times its probability. Comparing expected values helps decide about insurance, warranties and games, but it describes a long-run average, not one outcome.

Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5 · **Level:** Grade 12 Precalculus

What is a probability distribution?

A probability distribution lists every value a random variable X can take with its probability. Two rules must hold: each probability is between 0 and 1, and all the probabilities add to exactly 1. A *theoretical* distribution comes from equally likely outcomes, such as the sectors of a spinner. An *empirical* distribution comes from data, using relative frequencies.



An 8-sector spinner: four sectors pay \$0, two pay \$2, one pays \$5 and one pays \$10.

Payout x	\$0	\$2	\$5	\$10
$P(x)$	$\frac{4}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

How do you compute an expected value?

Expected value

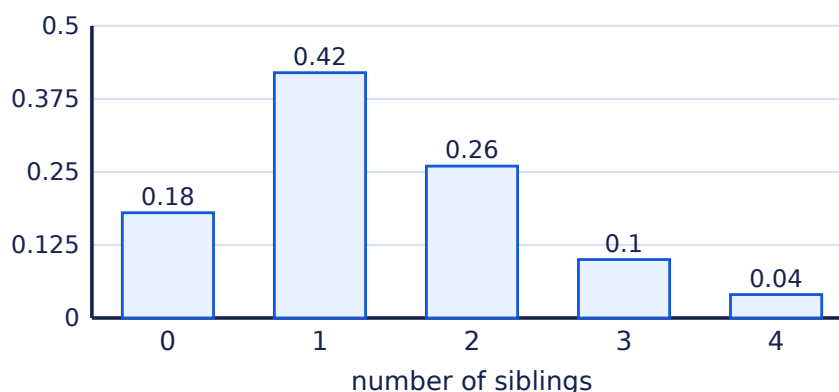
$E(X) = x_1p_1 + x_2p_2 + \dots + x_np_n$. It is the long-run average of X over many repetitions.

Worked example

For the spinner, $E(X) = 0(\frac{4}{8}) + 2(\frac{2}{8}) + 5(\frac{1}{8}) + 10(\frac{1}{8}) = \frac{19}{8} = \$2.375$. A spin pays about \$2.38 on average.

How do you build a distribution from data?

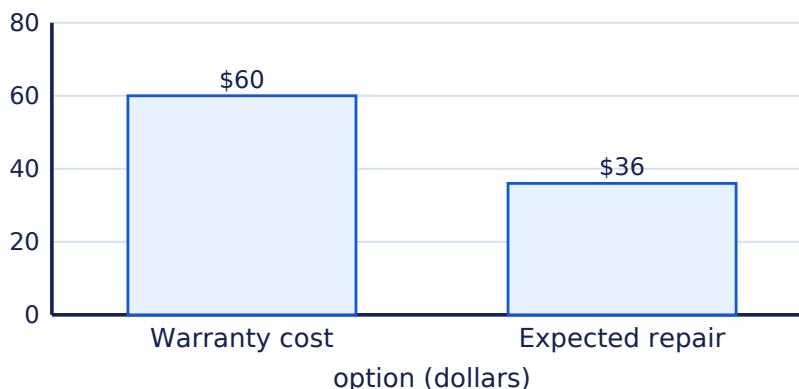
Count how often each value occurs and divide by the total. Fifty students reported their number of siblings: 0 siblings (9 students), 1 (21), 2 (13), 3 (5) and 4 (2). The relative frequencies 0.18, 0.42, 0.26, 0.10 and 0.04 form an empirical distribution, and $E(X) = 0(0.18) + 1(0.42) + 2(0.26) + 3(0.10) + 4(0.04) = 1.40$ siblings.



Bar chart.

How do you use expected value to make a decision?

Compute the expected value of each option and compare them. A game is *fair* when the expected net gain is 0. A \$60 warranty on a tablet covers a \$240 repair that has a 15% chance of being needed. The expected repair cost is $0.15(240) = \$36$, so on average the warranty costs \$24 more than it saves.



Bar chart.

Common mistake

Treating expected value as a guarantee. It is a long-run average; for a single event you may also weigh risk, such as an unaffordable repair.

Mathbot's tip

Always subtract the price of playing or buying. Expected winnings and expected net gain are different numbers.

Key points to remember

- A valid distribution has probabilities between 0 and 1 that sum to 1.
- Theoretical probabilities come from equally likely outcomes; empirical ones come from relative frequencies.
- $E(X) = \sum x \cdot P(x)$.
- Expected net gain = expected winnings – cost; a fair game has expected net gain 0.
- Expected value is a long-run average and ignores risk in a single trial.

Expected Value and Smart Decisions: frequently asked questions

What is the difference between theoretical and empirical probabilities?

Theoretical probabilities come from a model of equally likely outcomes, such as a fair spinner. Empirical probabilities come from observed data, found by dividing each count by the total number of observations.

How can I check a probability distribution?

Verify that every probability is between 0 and 1 and that the probabilities add to exactly 1. If the sum is not 1, either a probability is missing or a value was entered incorrectly.

What does a negative expected net gain mean?

Over many plays the player loses money on average. A game with an expected net gain of $-\$0.50$ loses about 50 cents per play in the long run, even if some plays win.

Why do insurance companies profit if the expected value favors them?

They insure thousands of customers, so the long-run average becomes reliable. The premium is set above the expected claim cost, which leaves an expected profit on every policy.

Should I always choose the option with the larger expected value?

Not always. Expected value ignores how bad a rare, costly outcome can be. A person may buy a warranty or insurance with a negative expected value to avoid a loss they could not afford.