

Expected Value and Smart Decisions

Grade 12 – Precalculus math · Test Answer Key · Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5

Use the scoring notes for partial credit. Most items award a point for a correct distribution or setup and a point for the expected value or the decision. Interpretation sentences must match the numbers.

Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5 · **Level:** Grade 12 Precalculus

Question 1 Missing Probability (2 points)

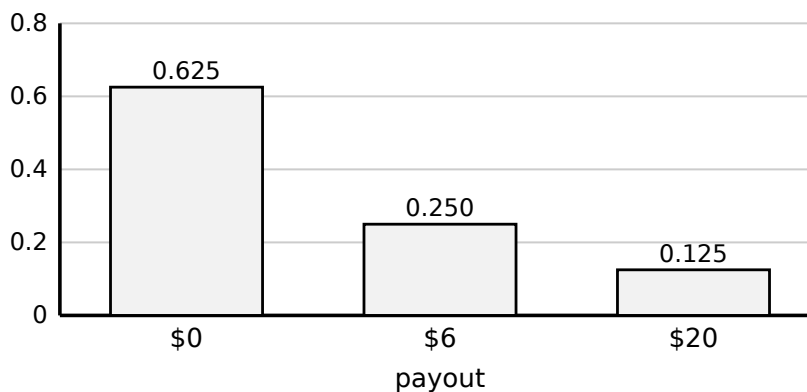
(a) $P(0) = 1 - (0.25 + 0.30 + 0.15) = 1 - 0.70 = 0.30.$

(b) $E(X) = (-1)(0.25) + 0(0.30) + 2(0.30) + 5(0.15) = -0.25 + 0 + 0.60 + 0.75 = 1.10.$

Answer: **(a) 0.30; (b) 1.10**

Scoring: 1 point for each part.

Question 2 Reading a Spinner (3 points)



Bar chart.

(a) $P(\$0) = \frac{5}{8}, P(\$6) = \frac{2}{8} = \frac{1}{4}, P(\$20) = \frac{1}{8}.$

(b) $E(X) = 0(\frac{5}{8}) + 6(\frac{2}{8}) + 20(\frac{1}{8}) = \frac{12}{8} + \frac{20}{8} = \frac{32}{8} = \$4.$

(c) Expected net gain = $4 - 5 = -\$1$, so the game is not fair; the player loses \$1 per spin on average.

Answer: **$E(X) = \$4$; expected net $-\$1$; not fair**

Scoring: 1 point for each part.

Question 3 Cars per Household (2 points)

Divide each count by 25: 0.08, 0.36, 0.40, 0.16. These add to 1.

$$E = 0(0.08) + 1(0.36) + 2(0.40) + 3(0.16) = 0.36 + 0.80 + 0.48 = 1.64.$$

Answer: **1.64 cars per household**

Scoring: 1 point for the distribution, 1 point for the expected value.

Question 4 Headphone Warranty (3 points)

(a) $0.10(150) = \$15$.

(b) The warranty costs $48 - 15 = \$33$ more than the expected repair cost.

(c) On average the warranty is not worth buying. A buyer who could not afford a surprise \$150 repair might still choose it to avoid that risk.

Answer: **expected repair cost \$15 versus \$48; do not buy on average**

Scoring: 1 point for each part.

Question 5 Insurer's Expected Profit (3 points)

(a) $0.04(3,500) = \$140$.

(b) $200 - 140 = \$60$.

(c) $1,500 \times 60 = \$90,000$.

Answer: **\$140; \$60; \$90,000**

Scoring: 1 point for each part.

Question 6 A Fair Prize (2 points)

(a) $E = 18(\frac{1}{6}) = \$3$.

(b) Let the prize be p . A fair game needs $p(\frac{1}{6}) = 6$, so $p = \$36$.

Answer: **(a) \$3; (b) \$36**

Scoring: 1 point for each part.

Question 7 Comparing Job Offers (3 points)

(a) $E = 4,000(0.3) + 2,000(0.5) + 500(0.2) = 1,200 + 1,000 + 100 = \$2,300$.

(b) Job B's expected pay is \$300 more than Job A's.

(c) Job B's pay varies a lot, and a person who needs a dependable income might choose Job A.

Answer: **Job B expected pay \$2,300 versus \$2,000**

Scoring: 1 point for each part.

Question 8 Larger of Two Dice (2 points)

Of the 16 equally likely pairs, $X = 1$ once, $X = 2$ three times, $X = 3$ five times and $X = 4$ seven times.

$$P(1) = \frac{1}{16}, P(2) = \frac{3}{16}, P(3) = \frac{5}{16}, P(4) = \frac{7}{16}.$$

$$E(X) = \frac{(1 + 6 + 15 + 28)}{16} = \frac{50}{16} = \frac{25}{8} = 3.125.$$

Answer: **$E(X) = \frac{25}{8} = 3.125$**

Scoring: 1 point for the distribution, 1 point for the expected value.