

Expected Value and Smart Decisions

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5

Each answer shows the distribution, checks that the probabilities add to 1 where needed, and computes the expected value term by term. A short sentence explains what the number means for the decision.

Standards: S-MD.A.3, S-MD.A.4, S-MD.B.5 · **Level:** Grade 12 Precalculus

Problem 1 Valid Distributions

(a) $0.2 + 0.3 + 0.4 + 0.2 = 1.1$, which is more than 1, so it is not valid.

(b) $0.25 + 0.25 + 0.30 + 0.20 = 1.00$ and every value is between 0 and 1, so it is valid.

Answer: **(a) not valid; (b) valid**

Problem 2 Missing Probability

The probabilities must add to 1: $0.15 + 0.35 + 0.20 = 0.70$, so $P(2) = 1 - 0.70 = 0.30$.

Answer: **0.30**

Problem 3 A Labeled Die

$$P(0) = \frac{2}{6} = \frac{1}{3}, P(1) = \frac{3}{6} = \frac{1}{2}, P(4) = \frac{1}{6}.$$

$$E(X) = 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{2}\right) + 4\left(\frac{1}{6}\right) = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} \approx 1.17.$$

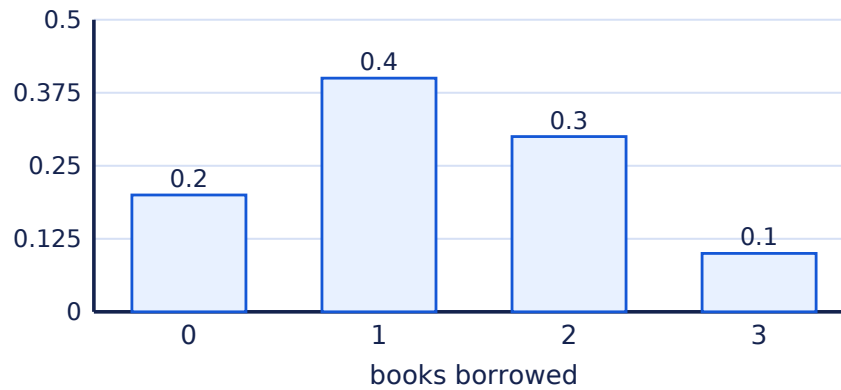
Answer: **$E(X) = \frac{7}{6} \approx 1.17$**

Problem 4 Expected Value from a Table

$$E(X) = 2(0.4) + 4(0.3) + 6(0.2) + 8(0.1) = 0.8 + 1.2 + 1.2 + 0.8 = 4.0.$$

Answer: **4.0**

Problem 5 Library Survey



Bar chart.

Divide by 30: $P(0) = 0.2$, $P(1) = 0.4$, $P(2) = 0.3$, $P(3) = 0.1$.

$$E = 0(0.2) + 1(0.4) + 2(0.3) + 3(0.1) = 0.4 + 0.6 + 0.3 = 1.3.$$

Answer: **1.3 books**

Problem 6 Is It Fair?

$$E(\text{winnings}) = 12\left(\frac{1}{4}\right) + 0\left(\frac{3}{4}\right) = \$3. \text{ Expected net gain} = 3 - 3 = \$0.$$

Answer: **yes, the game is fair (expected net gain \$0)**

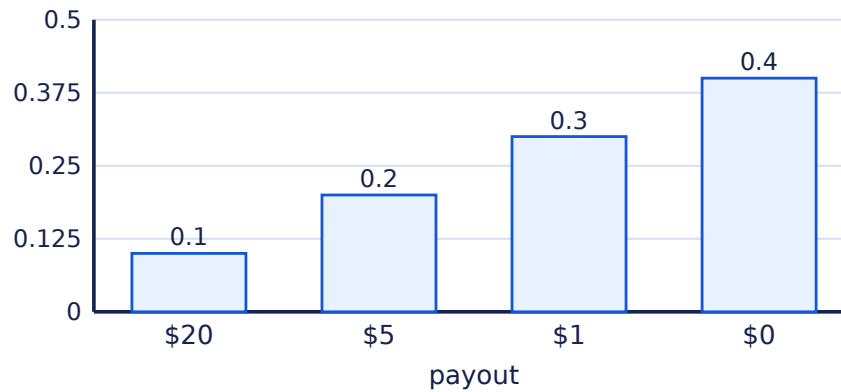
Problem 7 School Raffle

$$\text{Total prizes: } 300 + 2(50) = \$400. \text{ Expected winnings per ticket} = \frac{400}{200} = \$2.$$

$$\text{Expected net gain} = 2 - 5 = -\$3.$$

Answer: **\$2.00 expected winnings; -\$3.00 expected net gain**

Problem 8 Ten-Sector Spinner



Bar chart.

$$P(\$20) = 0.1, P(\$5) = 0.2, P(\$1) = 0.3, P(\$0) = 0.4.$$

$$E(\text{payout}) = 20(0.1) + 5(0.2) + 1(0.3) + 0 = 2 + 1 + 0.3 = \$3.30. \text{ Expected net gain} = 3.30 - 3.00 = \$0.30.$$

Answer: **expected net gain +\$0.30 per spin**

Problem 9 Free Throws

$$P(0) = 0.3 \times 0.3 = 0.09. P(2) = 0.7 \times 0.7 = 0.49. P(1) = 2(0.7)(0.3) = 0.42. \text{ The three add to } 1.$$

$$E(X) = 0(0.09) + 1(0.42) + 2(0.49) = 0.42 + 0.98 = 1.40.$$

Answer: **0.09, 0.42, 0.49; $E(X) = 1.4$ shots**

Problem 10 Insurance Policy

$$\text{Expected loss} = 0.03(2,500) = \$75.$$

The policy costs \$120 but covers an expected \$75, so the expected net result of buying is $75 - 120 = -\$45$.

Answer: **expected loss \$75; buying the policy costs \$45 more on average**

Problem 11 Speaker Warranty

$$\text{Expected repair cost} = 0.12(180) = \$21.60.$$

The warranty costs $55 - 21.60 = \$33.40$ more than the expected repair cost, so on average it is not worth buying.

Answer: **expected repair cost \$21.60 versus \$55 warranty**

Problem 12 Comparing Two Games

Game A: $5(0.3) - 2 = 1.50 - 2 = -\$0.50$. Game B: $3(0.6) - 2 = 1.80 - 2 = -\$0.20$.

Both lose money on average, but Game B loses less per play.

Answer: **A: $-\$0.50$; B: $-\$0.20$; Game B is better**

Problem 13 Televisions per Household

Relative frequencies: 0.05, 0.25, 0.45, 0.20, 0.05 (counts 3, 15, 27, 12, 3 divided by 60).

$$E = 0(0.05) + 1(0.25) + 2(0.45) + 3(0.20) + 4(0.05) = 0.25 + 0.90 + 0.60 + 0.20 = 1.95.$$

Answer: **1.95 televisions**

Problem 14 Two Four-Sided Dice

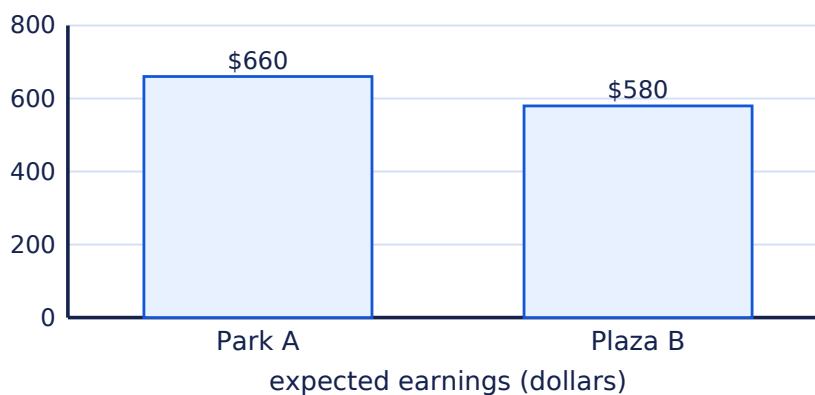
There are 16 equally likely pairs. The sums 2, 3, 4, 5, 6, 7, 8 occur 1, 2, 3, 4, 3, 2, 1 times.

x	2	3	4	5	6	7	8
$P(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$$E(X) = \frac{(2 + 6 + 12 + 20 + 18 + 14 + 8)}{16} = \frac{80}{16} = 5. \quad P(X \geq 6) = \frac{(3 + 2 + 1)}{16} = \frac{6}{16} = \frac{3}{8}.$$

Answer: **$E(X) = 5$ and $P(X \geq 6) = \frac{3}{8}$**

Problem 15 Food Truck Location



Bar chart.

Park A: $0.6(900) + 0.4(300) = 540 + 120 = \$660$.

Plaza B: $0.6(600) + 0.4(550) = 360 + 220 = \$580$.

Park A has the higher expected earnings by \$80. Plaza B has less variation, so it is the safer choice if the owner dislikes risk.

Answer: **Park A (\$660) beats Plaza B (\$580) on expected value**

Problem 16 Insurer's Profit

Expected claim per policy = $0.02(3,000) = \$60$.

Expected profit per policy = $90 - 60 = \$30$.

Expected total profit = $5,000 \times 30 = \$150,000$.

Answer: **\$60 per policy claim cost; \$30 profit per policy; \$150,000 in total**

Problem 17 Guessing on a Quiz

(a) $E = 4(\frac{1}{5}) + (-1)(\frac{4}{5}) = \frac{4}{5} - \frac{4}{5} = 0$, so guessing is a fair gamble.

(b) $E = 4(\frac{1}{3}) + (-1)(\frac{2}{3}) = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \approx 0.67$.

With three choices left the expected score is positive, so guessing helps on average. With five choices, guessing neither helps nor hurts.

Answer: **(a) 0; (b) $\frac{2}{3} \approx 0.67$, so guess in case (b)**

Problem 18 Screening Parts

Without screening: $0.04(25) = \$1.00$ per part in expected refunds.

With screening: \$0.50 per part and no refunds.

Screening saves $1.00 - 0.50 = \$0.50$ per part on average, so screening is recommended.

Answer: **\$1.00 versus \$0.50 per part; screen the parts**

Problem 19 Coin Flip Game

Heads 0, 1, 2, 3 have probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$, so the winnings \$0, \$2, \$4, \$6 have those probabilities.

$E(\text{winnings}) = 0(\frac{1}{8}) + 2(\frac{3}{8}) + 4(\frac{3}{8}) + 6(\frac{1}{8}) = \frac{24}{8} = \$3$.

Expected net gain = $3 - 4 = -\$1$.

Answer: **expected winnings \$3; expected net gain -\$1**

Problem 20 Fundraiser Drawing

(a) Total prizes: $1,000 + 4(150) + 20(25) = 1,000 + 600 + 500 = \$2,100$. Expected winnings per ticket = $\frac{2,100}{400} = \$5.25$. Expected net gain = $5.25 - 10 = -\$4.75$.

(b) Money collected: $400(10) = \$4,000$. Profit = $4,000 - 2,100 = \$1,900$.

Answer: **(a) \$5.25 expected winnings, -\$4.75 net; (b) \$1,900 profit**