

Domains, Composition and Inverse Functions Explained

Grade 12 – Precalculus math · Lesson · Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d

Quick answer: The domain of a function is the set of allowed inputs, composition feeds one function's output into another, and an inverse function undoes the original by swapping inputs and outputs. A function has an inverse only when it is one-to-one, which restricting the domain can guarantee.

Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · **Level:** Grade 12 Precalculus

How do you find the domain of a function?

The domain is the set of inputs a function can accept. For real-valued functions, only two things cause trouble: dividing by zero and taking an even root of a negative number. Write the answer in interval notation, where a square bracket keeps an endpoint and a parenthesis leaves it out.

Vocabulary

- **Domain:** every allowed input; **range:** every output that actually occurs.
- **One-to-one:** no two inputs share an output (passes the horizontal line test).
- **Composition:** $(f \circ g)(x) = f(g(x))$.
- **Inverse:** the function that undoes $f(x)$, written $f^{-1}(x)$.

Worked example: domain

For $f(x) = \sqrt{x - 3} \div (x - 7)$, the root needs $x - 3 \geq 0$, so $x \geq 3$, and the denominator forbids $x = 7$. The domain is **$[3, 7) \cup (7, \infty)$** .

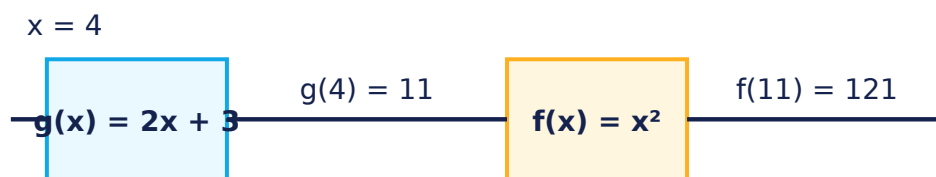


Domain of $\sqrt{x - 3} / (x - 7)$: $[3, 7) \cup (7, \infty)$.

How do you compose and decompose functions?

To evaluate $f(g(x))$, work from the inside out. The order matters: with $g(x) = 2x + 3$ and $f(x) = x^2$,

we get $(f \circ g)(x) = (2x + 3)^2$, but $(g \circ f)(x) = 2x^2 + 3$. To decompose $h(x) = \sqrt{(5x - 1)}$, name the inner piece $5x - 1$ and the outer piece $\sqrt{u}$. A composite is defined only where x is in the domain of the inner function and its output is in the domain of the outer one.



inner function first, then outer function

$$(f \circ g)(4) = f(g(4)) = 121$$

Composition feeds the output of the inner function into the outer function.

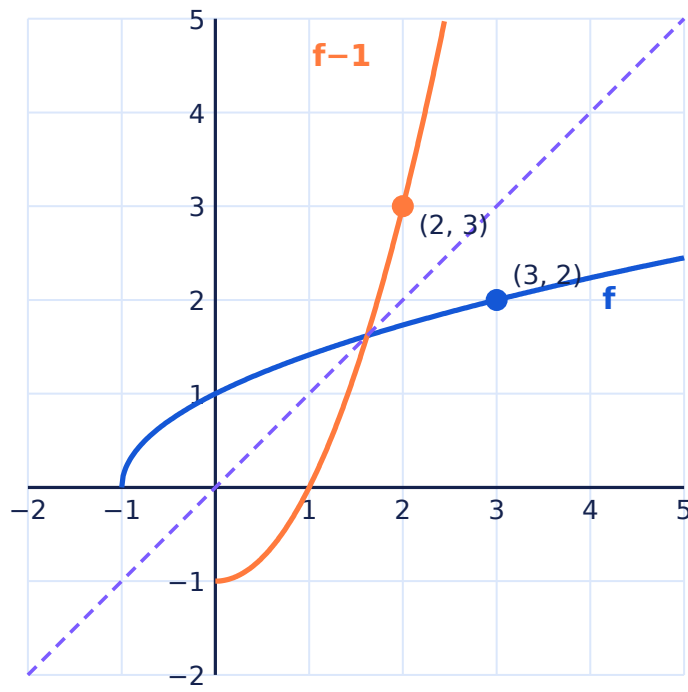
How do you find an inverse function?

Method: swap and solve

1. Write $y = f(x)$.
2. Swap x and y .
3. Solve the new equation for y .
4. Write $f^{-1}(x) = \text{that expression}$ and check that $f(f^{-1}(x)) = x$.

Worked example: inverse

Let $f(x) = \frac{(2x + 1)}{(x - 3)}$. Swapping gives $x = \frac{(2y + 1)}{(y - 3)}$, so $xy - 3x = 2y + 1$ and $y(x - 2) = 3x + 1$. Therefore $f^{-1}(x) = \frac{(3x + 1)}{(x - 2)}$. Domain and range trade places.



A function and its inverse are mirror images across the line $y = x$.

Tables and graphs work the same way: if $f(3) = 8$, then $f^{-1}(8) = 3$. A function that fails the horizontal line test has no inverse until its domain is restricted, for example x^2 on $x \geq 0$.

Common mistake

$f^{-1}(x)$ does not mean $1/f(x)$. The little -1 labels the inverse function; it is not an exponent.

Mathbot's tip

Test your inverse by plugging a number through both functions. If you do not land back where you started, an algebra step slipped.

Key points to remember

- Exclude zero denominators and negative numbers under even roots when finding a domain.
- $(f \circ g)(x) = f(g(x))$: evaluate the inner function first; order matters.
- To invert a function, swap x and y , then solve for y .
- The graph of an inverse is the reflection of the original across the line $y = x$.
- If $f(a) = b$ then $f^{-1}(b) = a$, and $f(f^{-1}(x)) = x$.

Domains, Composition and Inverse Functions Explained: frequently

asked questions

What is the domain of a function in interval notation?

It lists every allowed input as one or more intervals. Use brackets for included endpoints, parentheses for excluded ones, and the union symbol $\cup$ to join separate pieces, such as $(-\infty, 2) \cup (2, \infty)$.

Is $f(g(x))$ the same as $g(f(x))$?

Usually not. The inner function acts first, so swapping the order changes the result. For $f(x) = x + 5$ and $g(x) = 3x$, $f(g(x)) = 3x + 5$ but $g(f(x)) = 3x + 15$.

How can you tell whether a function has an inverse?

Apply the horizontal line test: if any horizontal line crosses the graph twice, two inputs share an output and the function is not one-to-one. Restrict the domain to a one-to-one piece to create an inverse.

Is the inverse $f^{-1}(x)$ equal to $1/f(x)$?

No. The reciprocal $1/f(x)$ divides 1 by the output, while $f^{-1}(x)$ undoes the function. For $f(x) = 2x$, the inverse is $x/2$ and the reciprocal is $1/(2x)$.

How do you restrict a domain to make a function invertible?

Keep one side of the turning point. For $f(x) = (x - 4)^2 + 1$, choosing $x \geq 4$ makes the graph strictly increasing, so every output has exactly one input.