

Domains, Composition and Inverse Functions Explained

Grade 12 – Precalculus math · Test Answer Key · Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d

Each solution lists the work and a scoring note. Award points for the idea and the algebra separately, so one arithmetic slip does not erase credit for a correct method.

Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · **Level:** Grade 12 Precalculus

Question 1 Domains in Interval Notation (2 points)

- a. $2x - 9 \geq 0$ gives $x \geq 9/2$: **$[9/2, \infty)$** .
 b. $x^2 - 16 = 0$ at $x = \pm 4$: **$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$** .

Scoring: 1 point per correct domain.

Question 2 Numerical Compositions (2 points)

- a. $g(2) = 2$, $f(2) = 4 + 4 = 8$.
 b. $f(-1) = 1 - 2 = -1$, $g(-1) = -3 - 4 = -7$.

Scoring: 1 point per value.

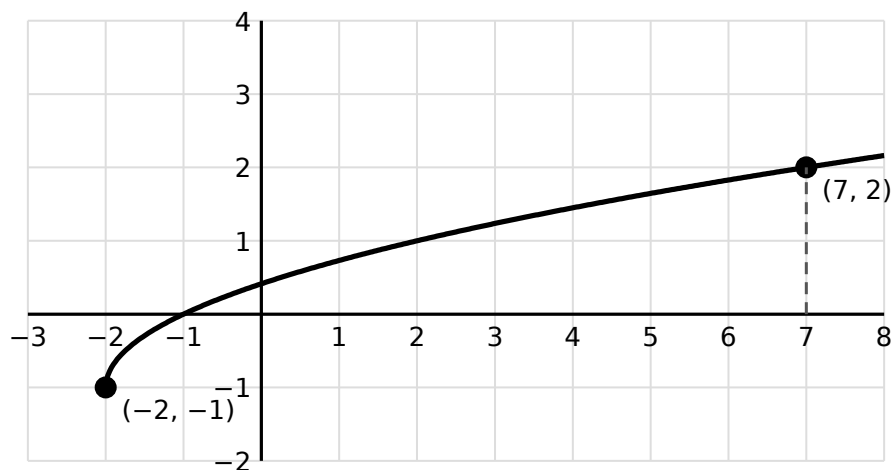
Question 3 Composite Expressions (2 points)

$$(f \circ g)(x) = (x + 3)^2 - 6(x + 3) = x^2 + 6x + 9 - 6x - 18 = x^2 - 9.$$

$$(g \circ f)(x) = x^2 - 6x + 3 = x^2 - 6x + 3.$$

Scoring: 1 point per composite.

Question 4 Domain, Range and Inverse from a Graph (2 points)



The point $(7, 2)$ gives $f^{-1}(2) = 7$.

- Domain $[-2, \infty)$, range $[-1, \infty)$.
- Since $f(7) = 2$, $f^{-1}(2) = 7$.

Scoring: 1 point for both intervals; 1 point for the inverse value.

Question 5 Find an Inverse Function (3 points)

Swap: $x = \frac{(4y + 3)}{(y - 2)}$. Then $xy - 2x = 4y + 3$, so $xy - 4y = 2x + 3$ and $y(x - 4) = 2x + 3$.

$$f^{-1}(x) = \frac{(2x + 3)}{(x - 4)}.$$

Scoring: 1 point for swapping, 1 point for clearing the denominator and collecting y , 1 point for the final formula.

Question 6 Verify Inverses (2 points)

$$f(g(x)) = \sqrt[3]{((x + 2)^3 - 5 + 5)} - 2 = (x + 2) - 2 = x.$$

$$g(f(x)) = (\sqrt[3]{(x + 5)})^3 - 5 = x + 5 - 5 = x. \text{ Both compositions give } x.$$

Scoring: 1 point per composition.

Question 7 Table of an Inverse (2 points)

- Swap the columns: $f^{-1}(7) = -1$, $f^{-1}(3) = 2$, $f^{-1}(0) = 5$, $f^{-1}(-4) = 9$.
- $f(f^{-1}(-4)) = -4$ and $f^{-1}(7) = -1$, so the sum is -5 .

Scoring: 1 point for the table; 1 point for the sum.

Question 8 Restrict a Parabola, Then Invert (3 points)

- a. $f(x) = (x - 3)^2 - 4$.
- b. Swap: $x = (y - 3)^2 - 4$, so $y - 3 = \sqrt{x + 4}$ (positive since $y \geq 3$): **$f^{-1}(x) = 3 + \sqrt{x + 4}$** .
- c. Domain **$[-4, \infty)$** , range **$[3, \infty)$** .

Scoring: 1 point per part.

Question 9 Sale Price with Tax (2 points)

- a. $C(p) = 1.07(0.80p) = \mathbf{0.856p}$.
- b. $0.856p = 64.20$ gives $p = 64.20 \div 0.856 = \mathbf{\$75}$.

Scoring: 1 point per part.