

Domains, Composition and Inverse Functions Explained

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d

Each answer shows every algebra step and ends with the result in bold. Compare your reasoning, not only the final value, and use the checks (plugging numbers through both functions) to catch slips in your own inverse formulas.

Standards: F-IF.A.1, F-BF.A.1c, F-BF.B.4b, F-BF.B.4c, F-BF.B.4d · **Level:** Grade 12 Precalculus

Problem 1 Evaluate with Function Notation

$$f(-3) = 2(9) - 5(-3) + 7 = 18 + 15 + 7 = \mathbf{40}.$$

$$f(4) = 2(16) - 20 + 7 = 32 - 20 + 7 = \mathbf{19}.$$

Problem 2 Domain Using Two Restrictions

(a) Need $4x + 9 \geq 0$, so $x \geq -9/4$. Domain: $[-9/4, \infty)$.

(b) Need $x + 6 \neq 0$, so $x \neq -6$. Domain: $(-\infty, -6) \cup (-6, \infty)$.

Problem 3 Domain and Range from a Graph

The curve starts at $x = -3$ (closed) and stops before $x = 5$ (open): domain $[-3, 5)$.

The highest output is 4. The lowest output is 0, reached at $(-3, 0)$, so the open dot does not matter: range $[0, 4]$.

Problem 4 Evaluate Two Compositions

$$g(2) = 9, \text{ so } f(9) = 81 - 3 = \mathbf{78}.$$

$$f(2) = 1, \text{ so } g(1) = \mathbf{5}.$$

Problem 5 Build Composite Functions

$$(f \circ g)(x) = 6(x^2 - 4) + 1 = \mathbf{6x^2 - 23}.$$

$$(g \circ f)(x) = (6x + 1)^2 - 4 = 36x^2 + 12x + 1 - 4 = \mathbf{36x^2 + 12x - 3}.$$

Problem 6 Inverse of a Linear Function

Swap: $x = \frac{5}{4}y + 3$. Subtract 3: $x - 3 = \frac{5}{4}y$. Multiply by $\frac{4}{5}$: $y = \frac{4}{5}(x - 3)$.

$$f^{-1}(x) = \frac{(4x - 12)}{5}.$$

Problem 7 Reading Inverse Values from a Table

$f(5) = 15$, so $f^{-1}(15) = \mathbf{5}$. $f(11) = 30$, so $f^{-1}(30) = \mathbf{11}$.

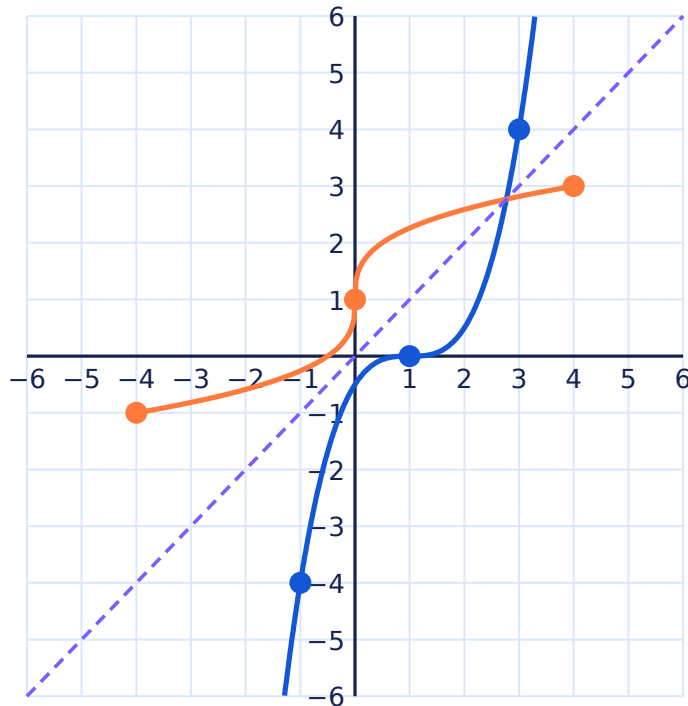
$f(f^{-1}(22)) = \mathbf{22}$ because a function and its inverse cancel.

The output 10 does not appear in the table, so $f^{-1}(10)$ **cannot be found** from these data.

Problem 8 Inverse Values from a Graph

Swap the coordinates: (4, 3), (0, 1) and (-4, -1) lie on the inverse.

$$f^{-1}(4) = \mathbf{3}, f^{-1}(0) = \mathbf{1}, f^{-1}(-4) = \mathbf{-1}.$$



Each point of f reflects across $y = x$ onto the inverse.

Problem 9 Domain of a Root over a Binomial

The root requires $x + 5 \geq 0$, so $x \geq -5$. The denominator requires $x \neq 2$, and 2 lies inside that range.

Domain: $[-5, 2) \cup (2, \infty)$.

Problem 10 Verify Inverse Functions

$$f(g(x)) = 2 \cdot \frac{(x-1)}{2} + 1 = x - 1 + 1 = x.$$

$$g(f(x)) = \sqrt[3]{(2x^3)/2} = \sqrt[3]{x^3} = x.$$

Both compositions equal x , so **they are inverses**.

Problem 11 Decompose a Composite Function

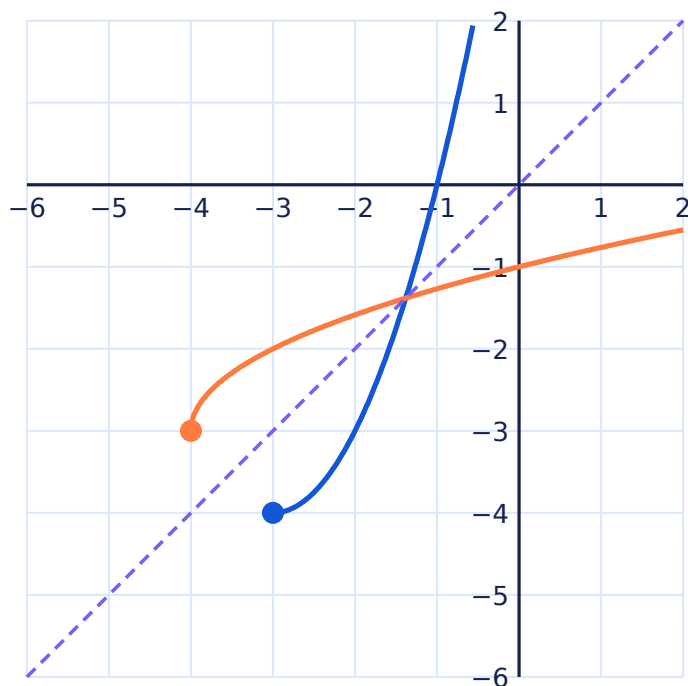
(a) Inner $g(x) = 5x - 2$, outer $f(u) = u^4$: **$h = f \circ g$** .

(b) Inner $g(x) = x^2 + 9$, outer $f(u) = 1/u$: **$k = f \circ g$** .

Problem 12 Restrict, Then Invert

On $x \geq -3$ the graph rises, so it is one-to-one. Swap: $x = (y + 3)^2 - 4$, so $y + 3 = \sqrt{x + 4}$ (positive root because $y \geq -3$).

$$\mathbf{f^{-1}(x) = \sqrt{(x + 4)} - 3}$$
, domain $[-4, \infty)$, range $[-3, \infty)$.



The restricted function and its inverse mirror each other.

Problem 13 Inverse of a Rational Function

Swap: $x = \frac{(y+1)}{(y-4)}$. Then $xy - 4x = y + 1$, so $y(x - 1) = 4x + 1$.

$f^{-1}(x) = \frac{(4x+1)}{(x-1)}$, and $x = 1$ is excluded (1 is not in the range of f).

Problem 14 Domain of a Composite

$$(f \circ g)(x) = \sqrt{(x^2 - 10) - 1} = \sqrt{(x^2 - 11)}.$$

Need $x^2 \geq 11$, so $|x| \geq \sqrt{11}$. Domain: $(-\infty, -\sqrt{11}] \cup [\sqrt{11}, \infty)$.

Problem 15 Horizontal Line Test and Restriction

(a) $x^3 - 3x = x(x^2 - 3) = 0$ gives $x = 0, \sqrt{3}, -\sqrt{3}$. So $g(0) = g(\sqrt{3}) = g(-\sqrt{3}) = 0$, and g is not one-to-one.

(b) The turning points are at $x = \pm 1$, so g is one-to-one on $[1, \infty)$ (also on $(-\infty, -1]$ or $[-1, 1]$).

Problem 16 Taxi Fare and Its Inverse

Swap and solve: $m = (C - 2.50) \div 1.75$, that is $f^{-1}(x) = \frac{(x - 2.50)}{1.75}$. It converts a fare in dollars to miles driven.

$$f^{-1}(23.50) = 21 \div 1.75 = \mathbf{12 \text{ miles.}}$$

Problem 17 Lemonade Stand Revenue

$$r(c(t)) = 1.75(12t + 8) - 20 = 21t + 14 - 20 = \mathbf{21t - 6.}$$

$$\text{At } t = 5: 105 - 6 = \mathbf{\$99.}$$

$$\text{Solve } 21t - 6 = 162: t = 168 \div 21 = \mathbf{8 \text{ hours.}}$$

Problem 18 Spreading Ripple Area

$$A(r(t)) = \pi(4 + 3t)^2.$$

$$\text{At } t = 2: \pi(10)^2 = \mathbf{100\pi \approx 314.16 \text{ ft}^2.}$$

$$(4 + 3t)^2 = 169 \text{ gives } 4 + 3t = 13 \text{ (radius is positive), so } t = \mathbf{3 \text{ seconds.}}$$

Problem 19 Cabin Cost Split

From $s = \frac{480}{(n+4)}$ we get $n + 4 = \frac{480}{s}$, so $n = \frac{480}{s} - 4$. Using x as the input: $f^{-1}(x) = \frac{480}{x} - 4$, the number of students for a share of x dollars.

$$f^{-1}(32) = 15 - 4 = \mathbf{11 \text{ students.}}$$

Problem 20 Restrict, Invert and Evaluate

$$h(x) = 2(x^2 - 1) + 3 = \mathbf{2x^2 + 1.}$$
 On $x \geq 0$ it is one-to-one.

$$\text{Swap: } x = 2y^2 + 1, \text{ so } y = \sqrt{(x-1)/2} \text{ (positive root since } y \geq 0). \text{ Hence } \mathbf{h^{-1}(x) = \sqrt{(x-1)/2}.}$$

$$h^{-1}(19) = \sqrt{9} = \mathbf{3.}$$
 Check: $h(3) = 19$.