

Inverse Trigonometric Functions and Trig Equations

Grade 12 – Precalculus math · Lesson · Standards: F-TF.B.6, F-TF.B.7

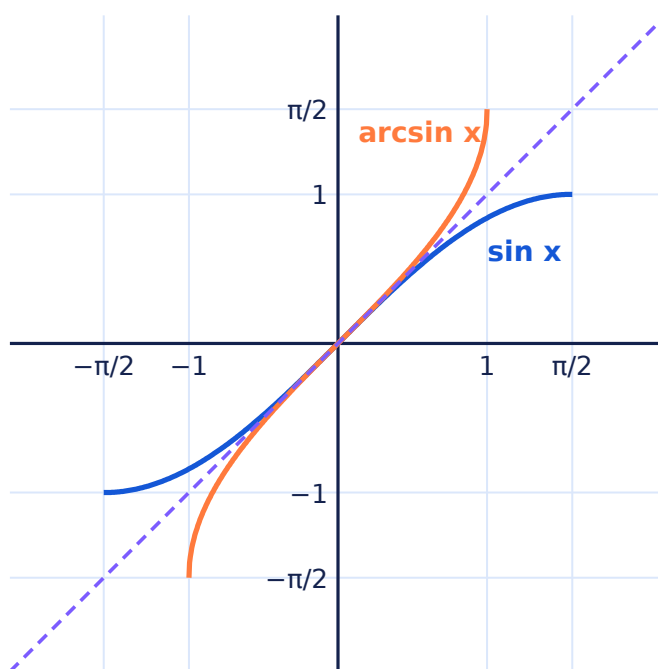
Quick answer: Inverse trigonometric functions return the one angle in a restricted interval that has a given sine, cosine or tangent: arcsin gives angles in $[-\pi/2, \pi/2]$, arccos in $[0, \pi]$, and arctan in $(-\pi/2, \pi/2)$. Solving trig equations means finding all angles in one period and adding multiples of the period.

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Why do inverse trigonometric functions need restricted domains?

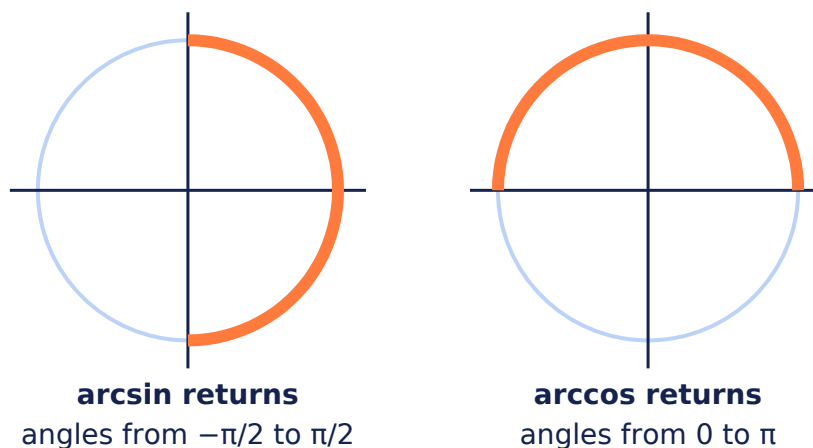
Sine, cosine and tangent repeat, so they fail the horizontal line test. To invert them we keep one piece on which each is one-to-one. The inverse then returns exactly one angle.

Inverse	Input (domain)	Output (range)	Based on
$\arcsin x$	$[-1, 1]$	$[-\pi/2, \pi/2]$	sin on $[-\pi/2, \pi/2]$
$\arccos x$	$[-1, 1]$	$[0, \pi]$	cos on $[0, \pi]$
$\arctan x$	all real numbers	$(-\pi/2, \pi/2)$	tan on $(-\pi/2, \pi/2)$



Restricting sine to $[-\pi/2, \pi/2]$ makes it invertible; arcsin is its reflection.

The notation $\sin^{-1} x$ means $\arcsin x$, not $1/\sin x$. The value $\arcsin(1/2) = \pi/6$ asks: which angle in $[-\pi/2, \pi/2]$ has sine $1/2$?



The highlighted arcs are the possible outputs of arcsin and arccos.

Common mistake

Assuming $\arcsin(\sin x) = x$ for every x . That is true only when x is in the range $[-\pi/2, \pi/2]$. For example $\arcsin(\sin 5\pi/6) = \arcsin(1/2) = \pi/6$, not $5\pi/6$.

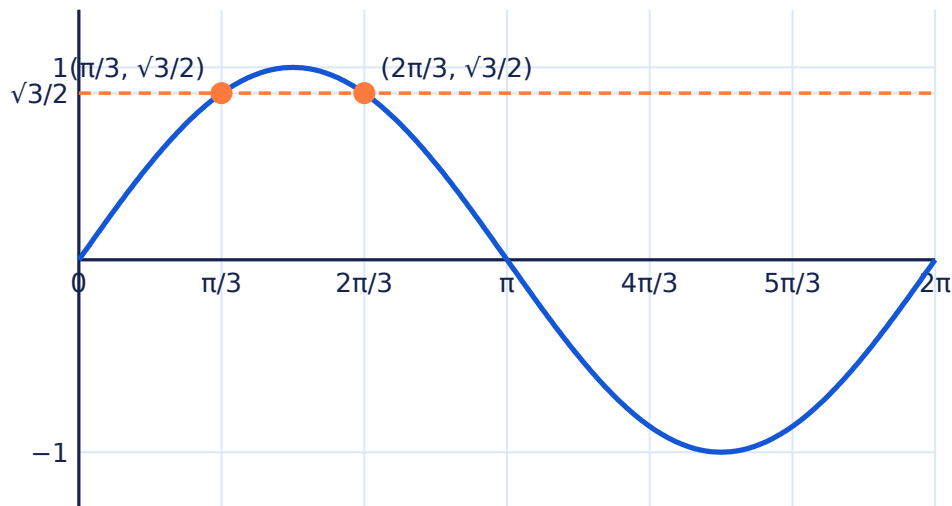
How do you solve a trigonometric equation?

Method: find every solution

1. Isolate the trigonometric expression (factor or use a quadratic form if needed).
2. Use the inverse function and symmetry to list all solutions in one period: $[0, 2\pi)$ for sine and cosine, $[0, \pi)$ for tangent.
3. For all real solutions add $2\pi k$ (sine, cosine) or πk (tangent), where k is an integer.
4. For a restricted interval, keep only the values that fit.

Worked example

Solve $2 \sin x = \sqrt{3}$. Then $\sin x = \sqrt{3}/2$, with reference angle $\pi/3$ and sine positive in Quadrants I and II. On $[0, 2\pi)$: **$x = \pi/3$ and $2\pi/3$** . In general: **$x = \pi/3 + 2\pi k$ or $2\pi/3 + 2\pi k$** .



The equation $\sin x = \sqrt{3}/2$ has two solutions in $[0, 2\pi)$.

When the number is not special, use technology in radian mode. If $\cos x = 0.3$, the calculator gives $\arccos 0.3 \approx 1.2661$, and symmetry gives the second solution $2\pi - 1.2661 \approx 5.0171$. In a word problem, check that each solution fits the context before reporting it.

Mathbot's tip

Never divide both sides by a trigonometric factor such as $\cos x$. Move everything to one side and factor, so no solutions disappear.

Key points to remember

- $\arcsin$ has range $[-\pi/2, \pi/2]$; $\arccos$ has range $[0, \pi]$; $\arctan$ has range $(-\pi/2, \pi/2)$.
- $\sin^{-1} x$ means $\arcsin x$, not $1/\sin x$.
- $\arcsin(\sin x) = x$ only when x is in $[-\pi/2, \pi/2]$.
- General solutions add $2\pi k$ for sine and cosine and πk for tangent.
- Factor trigonometric equations instead of dividing by a trig factor.

Inverse Trigonometric Functions and Trig Equations: frequently asked questions

Why do inverse trigonometric functions need restricted domains?

Sine, cosine and tangent are periodic, so many inputs share the same output and the functions are not one-to-one. Restricting to one piece, such as sine on $[-\pi/2, \pi/2]$, makes an inverse function possible.

Is $\sin^{-1}(x)$ the same as $1/\sin(x)$?

No. The notation $\sin^{-1}(x)$ denotes the inverse function $\arcsin(x)$, which returns an angle. The

reciprocal $1/\sin(x)$ is the cosecant. Check the context to avoid mixing them up.

Why is $\arcsin(\sin x)$ not always equal to x ?

The output of $\arcsin$ must lie in $[-\pi/2, \pi/2]$. If x is outside that interval, $\arcsin$ returns the angle in that interval with the same sine, so $\arcsin(\sin(5\pi/6)) = \pi/6$.

How do you find all solutions of a trigonometric equation?

Solve for all angles in one period using the inverse function and symmetry, then add multiples of the period: $2\pi k$ for sine and cosine, πk for tangent. Restrict the list if an interval is given.

How do you solve a trigonometric equation with a calculator?

Set the calculator to radian mode, apply the inverse function to get one solution, and use symmetry to get the others in the period. Round only at the end and check that each answer fits the situation.