

Inverse Trigonometric Functions and Trig Equations

Grade 12 – Precalculus math · Test Answer Key · Standards: F-TF.B.6, F-TF.B.7

Each solution shows the work with a scoring note. Award points for the reference angle, for each correct branch of solutions, and for the general form separately, so a missed second solution costs only one point.

Standards: F-TF.B.6, F-TF.B.7 · **Level:** Grade 12 Precalculus

Question 1 Exact arcsin and arccos (2 points)

(a) The angle in $[-\pi/2, \pi/2]$ with sine $-\sqrt{2}/2$ is $-\pi/4$.

(b) The angle in $[0, \pi]$ with cosine $-1/2$ is $2\pi/3$.

Scoring: 1 point per value.

Question 2 arctan and Ranges (2 points)

(a) The angle in $(-\pi/2, \pi/2)$ with tangent $\sqrt{3}$ is $\pi/3$.

(b) Domain **all real numbers**; range $(-\pi/2, \pi/2)$.

Scoring: 1 point per part.

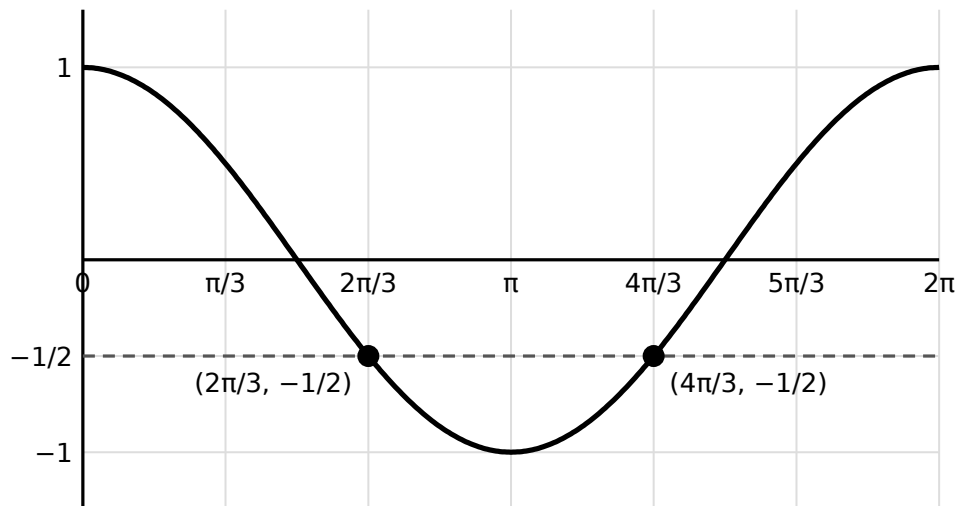
Question 3 Compositions (2 points)

(a) Opposite 8, hypotenuse 17, adjacent 15: **15/17**.

(b) $\sin(4\pi/3) = -\sqrt{3}/2$ and $\arcsin(-\sqrt{3}/2) = -\pi/3$.

Scoring: 1 point per part.

Question 4 Read Solutions from a Graph (2 points)



The solutions are $2\pi/3$ and $4\pi/3$.

- The line meets the curve **two** times.
- Cosine is $-1/2$ with reference angle $\pi/3$ in Quadrants II and III: **$x = 2\pi/3$ and $4\pi/3$** .

Scoring: 1 point per part.

Question 5 A Quadratic in Sine (3 points)

Factor: $(2 \sin x + 1)(\sin x + 1) = 0$.

$\sin x = -1/2$ gives $x = 7\pi/6$ and $11\pi/6$. $\sin x = -1$ gives $x = 3\pi/2$.

$x = 7\pi/6, 3\pi/2, 11\pi/6$.

Scoring: 1 point for the factorization; 1 point for each correct branch.

Question 6 Tangent of a Double Angle (3 points)

$\tan u = \sqrt{3}$ when $u = \pi/3 + k\pi$, so $2x = \pi/3 + k\pi$ and $x = \pi/6 + k\pi/2$.

On $[0, 2\pi)$: **$x = \pi/6, 2\pi/3, 7\pi/6, 5\pi/3$** . General: **$x = \pi/6 + k\pi/2$** .

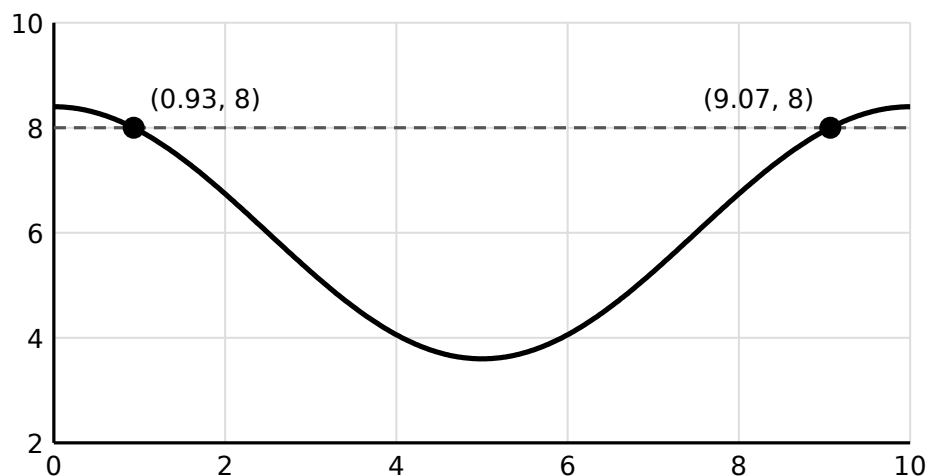
Scoring: 1 point for the general form of the angle; 1 point for the four solutions; 1 point for the general solution.

Question 7 Calculator Solution (2 points)

$\cos x = 0.45$. $\arccos 0.45 \approx \mathbf{1.1040}$. The second solution is $2\pi - 1.1040 \approx \mathbf{5.1792}$.

Scoring: 1 point for each solution.

Question 8 Wave Buoy Height (3 points)



The buoy is 8 ft high twice per period.

- a. $6 + 2.4\cos(\pi t/5) = 8$, so $\cos(\pi t/5) = 2/2.4 \approx 0.8333$.
- b. $\pi t/5 = \arccos(0.8333) \approx 0.5857$, so **$t \approx 0.93$ s**.
- c. By symmetry $t = 10 - 0.93 \approx$ **9.07 s**.

Scoring: 1 point per part.

Question 9 A False Identity (1 point)

$\sin \pi = 0$, and $\arcsin 0 = 0$, not π . The output of $\arcsin$ must lie in $[-\pi/2, \pi/2]$, and π is outside that interval.

Scoring: 1 point for a correct explanation with the numbers.