

Inverse Trigonometric Functions and Trig Equations

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-TF.B.6, F-TF.B.7

Each answer shows the reference angle or the inverse function value, then the symmetric solutions, with the final result in bold. Compare your list with the graph and check that every solution lies in the requested interval.

Standards: F-TF.B.6, F-TF.B.7 · **Level:** Grade 12 Precalculus

Problem 1 Evaluate arcsin

The answer must lie in $[-\pi/2, \pi/2]$.

(a) $\pi/3$. (b) $-\pi/6$. (c) **0**. (d) $-\pi/2$.

Problem 2 Evaluate arccos

The answer must lie in $[0, \pi]$.

(a) $\pi/3$. (b) **$5\pi/6$** . (c) π . (d) $\pi/2$.

Problem 3 Evaluate arctan

The answer must lie in $(-\pi/2, \pi/2)$.

(a) $\pi/4$. (b) $-\pi/3$. (c) **0**. (d) $\pi/6$.

Problem 4 Domain and Range from a Graph

The curve begins at $x = -1$ and ends at $x = 1$: domain **$[-1, 1]$** .

Its lowest height is $-\pi/2$ and its highest is $\pi/2$: range **$[-\pi/2, \pi/2]$** . At $x = 0$ the output is **0**.

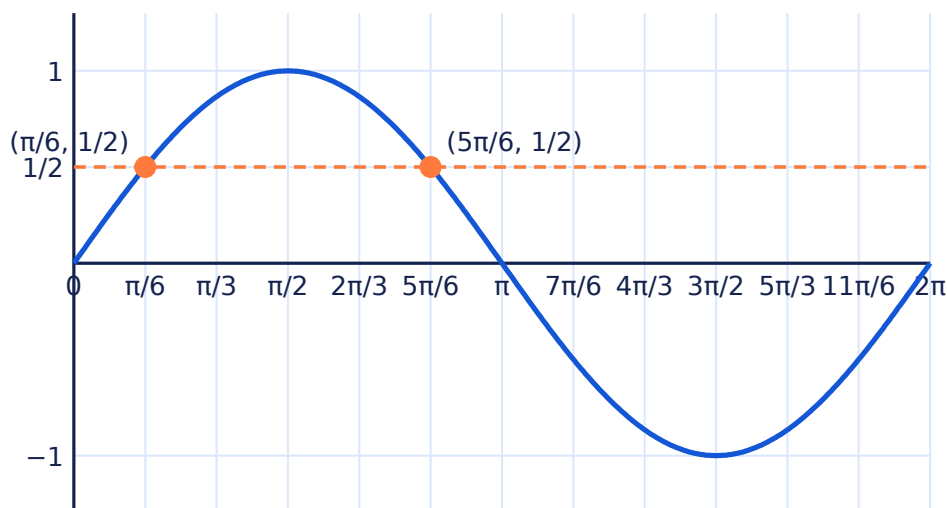
Problem 5 Why Restrict the Domain?

$\sin(\pi/6) = 1/2$ and $\sin(5\pi/6) = 1/2$. One output comes from two inputs, so sine is not one-to-one on $[0, 2\pi]$ and a single inverse cannot choose between them.

Restrict to **$[-\pi/2, \pi/2]$** , where sine is increasing.

Problem 6 Solve Three Basic Equations

- (a) Reference angle $\pi/6$, sine positive in Quadrants I and II: $x = \pi/6, 5\pi/6$.
- (b) Cosine is negative in Quadrants II and III with reference $\pi/4$: $x = 3\pi/4, 5\pi/4$.
- (c) Tangent is 1 at $\pi/4$ and every π after: $x = \pi/4, 5\pi/4$.



The two solutions of $\sin x = 1/2$.

Problem 7 Compositions with Triangles

- (a) Let $\theta = \arccos(8/17)$; the adjacent side is 8, the hypotenuse 17, so the opposite side is 15: **15/17**.
- (b) Opposite 7, hypotenuse 25, adjacent 24: **24/25**.
- (c) θ is in Quadrant IV (arcsin range), cosine is positive $12/13$, so $\tan = (-5/13) \div (12/13) = -5/12$.
- (d) Opposite 12, adjacent 5, hypotenuse 13: **12/13**.

Problem 8 Careful with $\arcsin(\sin x)$

- (a) $\sin(5\pi/6) = 1/2$ and $\arcsin(1/2) = \pi/6$.
- (b) $\cos(-\pi/3) = 1/2$ and $\arccos(1/2) = \pi/3$.
- (c) $\tan(3\pi/4) = -1$ and $\arctan(-1) = -\pi/4$.
- (d) $\cos(7\pi/4) = \sqrt{2}/2$ and $\arccos(\sqrt{2}/2) = \pi/4$.

Problem 9 Equation Solved in General

$\sin x = \sqrt{3}/2$, reference angle $\pi/3$, sine positive in Quadrants I and II.

On $[0, 2\pi)$: $x = \pi/3, 2\pi/3$. General: $x = \pi/3 + 2\pi k$ or $2\pi/3 + 2\pi k$, k an integer.

Problem 10 A Quadratic in Cosine

$\cos^2 x = 3/4$, so $\cos x = \pm\sqrt{3}/2$.

$\cos x = \sqrt{3}/2$ gives $x = \pi/6, 11\pi/6$; $\cos x = -\sqrt{3}/2$ gives $x = 5\pi/6, 7\pi/6$.

$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$.

Problem 11 Equation with a Multiple Angle Input

$\sin u = \sqrt{3}/2$ when $u = \pi/3 + 2\pi k$ or $2\pi/3 + 2\pi k$. With $u = 3x$: $x = \pi/9 + 2\pi k/3$ or $2\pi/9 + 2\pi k/3$.

On $[0, 2\pi)$: $x = \pi/9, 2\pi/9, 7\pi/9, 8\pi/9, 13\pi/9, 14\pi/9$. General: $x = \pi/9 + 2\pi k/3$ or $2\pi/9 + 2\pi k/3$.

Problem 12 Use a Calculator for $\cos x = 0.3$

In radian mode, $\arccos 0.3 \approx 1.2661$. By symmetry the second solution is $2\pi - 1.2661 \approx 5.0171$.

Problem 13 Negative Sine Value

$\arcsin(-0.45) \approx -0.4668$, which is in Quadrant IV but negative, so convert. The solutions lie in Quadrants III and IV.

Quadrant III: $\pi + 0.4668 \approx 3.6084$. Quadrant IV: $2\pi - 0.4668 \approx 5.8164$.

Problem 14 Tangent Equation with Technology

$\tan x = -5/3$. $\arctan(-5/3) \approx -1.0304$, which is outside $[0, 2\pi)$, so add π and 2π .

$x \approx 2.1112$ and 5.2528 .

Problem 15 Factor Instead of Dividing

Move terms: $\cos x(2 \sin x - 1) = 0$. Then $\cos x = 0$ gives $x = \pi/2, 3\pi/2$, and $\sin x = 1/2$ gives x

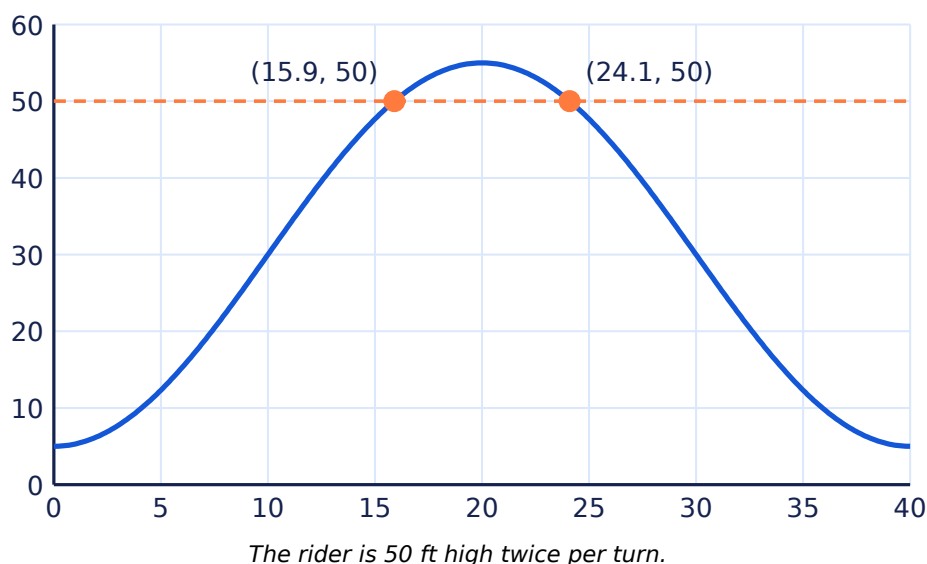
$= \pi/6, 5\pi/6$.

$x = \pi/6, \pi/2, 5\pi/6, 3\pi/2$. Dividing by $\cos x$ would remove the solutions where $\cos x = 0$, namely $\pi/2$ and $3\pi/2$.

Problem 16 Ferris Wheel at 50 Feet

Solve $30 - 25\cos(\pi t/20) = 50$: $\cos(\pi t/20) = -0.8$. Then $\pi t/20 = \arccos(-0.8) \approx 2.4981$, so $t \approx$ **15.9 s**.

By symmetry the second time is $40 - 15.9 \approx$ **24.1 s**.



Problem 17 Tide Window

Solve $3.5 \sin(\pi t/6) \geq 2$: $\sin(\pi t/6) \geq 0.5714$. Then $\arcsin 0.5714 \approx 0.6082$, so the first crossing is $\pi t/6 = 0.6082$, giving $t \approx 1.16$. The second is $\pi t/6 = \pi - 0.6082$, giving $t \approx 4.84$.

The depth is at least 6.5 ft for **$1.16 \leq t \leq 4.84$** , about **3.7 hours**.

Problem 18 Wheelchair Ramp Angles

(a) $\sin \theta = 1.1/14$, so $\theta = \arcsin(0.0786) \approx$ **4.51°** .

(b) $\tan \theta = 1/12$, so $\theta = \arctan(1/12) \approx$ **4.76°** .

(c) $h = 20 \sin(4.76^\circ) \approx$ **1.66 feet**.

Problem 19 Swinging Pendulum

Period = $2\pi \div (5\pi/4) = 1.6$ s. Solve $\cos(5\pi t/4) = 0.2/0.35 \approx 0.5714$: $5\pi t/4 = \arccos(0.5714) \approx$
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0.9626, so $t \approx \mathbf{0.245 \text{ s}}$.

The other solution is $5\pi t/4 = 2\pi - 0.9626$, so $t \approx \mathbf{1.355 \text{ s}}$.

Problem 20 Quadratic in Sine with an Impossible Factor

Factor: $(2 \sin x - 1)(\sin x + 2) = 0$.

$\sin x = -2$ is impossible because sine never goes below -1 . $\sin x = 1/2$ gives $\mathbf{x = \pi/6, 5\pi/6}$.

General: $\mathbf{x = \pi/6 + 2\pi k \text{ or } 5\pi/6 + 2\pi k}$.