

Inverse Trigonometric Functions and Trig Equations

Grade 12 – Precalculus math · Worksheet · Standards: F-TF.B.6, F-TF.B.7

Name: _____ Date: _____ Class: _____

These 20 problems cover the restricted domains of sine, cosine and tangent, the values of arcsin, arccos and arctan, compositions, and solving trigonometric equations on an interval and in general. They train standards F-TF.B.6 and F-TF.B.7. The last six problems use technology and context: a Ferris wheel, tides, a ramp, a pendulum and a factoring trap.

Standards: F-TF.B.6, F-TF.B.7 · **Level:** Grade 12 Precalculus

Problem 1 Evaluate arcsin (easy)

Find the exact value in radians. (a) $\arcsin(\sqrt{3}/2)$ (b) $\arcsin(-1/2)$ (c) $\arcsin 0$ (d) $\arcsin(-1)$

Problem 2 Evaluate arccos (easy)

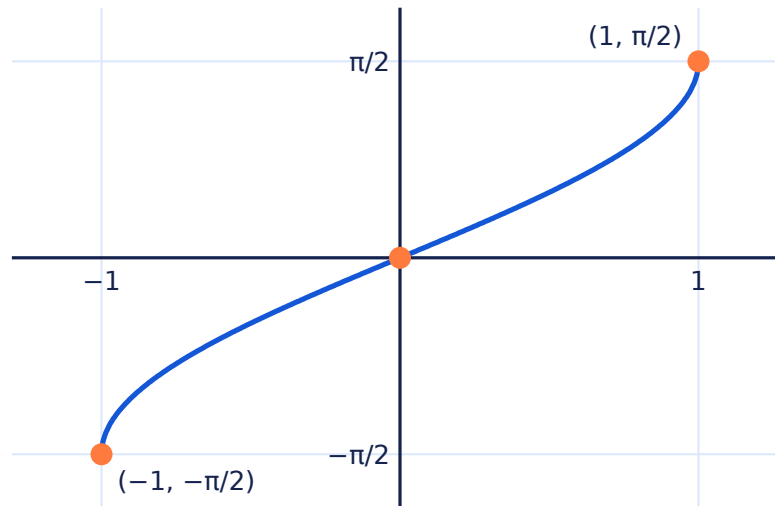
Find the exact value in radians. (a) $\arccos(1/2)$ (b) $\arccos(-\sqrt{3}/2)$ (c) $\arccos(-1)$ (d) $\arccos 0$

Problem 3 Evaluate arctan (easy)

Find the exact value in radians. (a) $\arctan 1$ (b) $\arctan(-\sqrt{3})$ (c) $\arctan 0$ (d) $\arctan(\sqrt{3}/3)$

Problem 4 Domain and Range from a Graph (easy)

The graph shows $y = \arcsin x$. State its domain and range, and give the output when $x = 0$.



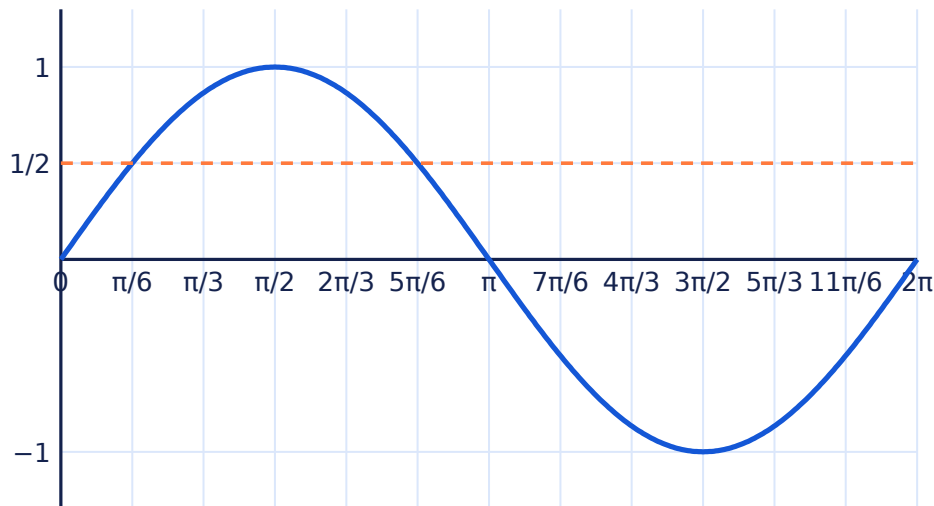
Graph of $y = \arcsin x$.

Problem 5 Why Restrict the Domain? (easy)

Name two inputs in $[0, 2\pi]$ for which $\sin x = 1/2$, and explain why that means sine has no inverse on $[0, 2\pi]$. Then give the standard interval used for $\arcsin$.

Problem 6 Solve Three Basic Equations (easy)

Solve on $[0, 2\pi)$. (a) $\sin x = 1/2$ (see the graph) (b) $\cos x = -\sqrt{2}/2$ (c) $\tan x = 1$



The line $y = 1/2$ crosses the sine curve twice.

Problem 7 Compositions with Triangles (medium)

Find the exact value. (a) $\sin(\arccos(8/17))$ (b) $\cos(\arcsin(7/25))$ (c) $\tan(\arcsin(-5/13))$ (d) $\sin(\arctan(12/5))$

Problem 8 Careful with arcsin(sin x) (medium)

Evaluate. (a) $\arcsin(\sin(5\pi/6))$ (b) $\arccos(\cos(-\pi/3))$ (c) $\arctan(\tan(3\pi/4))$ (d) $\arccos(\cos(7\pi/4))$

Problem 9 Equation Solved in General (medium)

Solve $2 \sin x = \sqrt{3}$ on $[0, 2\pi)$ and then write the general solution.

Problem 10 A Quadratic in Cosine (medium)

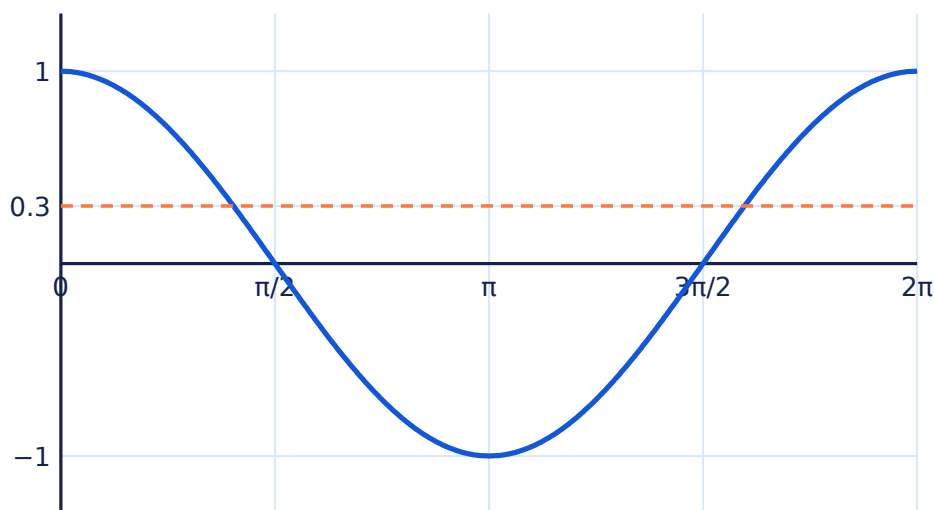
Solve $4\cos^2 x - 3 = 0$ on $[0, 2\pi)$.

Problem 11 Equation with a Multiple Angle Input (medium)

Solve $\sin(3x) = \sqrt{3}/2$ on $[0, 2\pi)$ and give the general solution.

Problem 12 Use a Calculator for $\cos x = 0.3$ (medium)

Solve $\cos x = 0.3$ on $[0, 2\pi)$. Give the solutions to four decimal places. The graph shows the line $y = 0.3$.



Two crossings of $y = 0.3$.

Problem 13 Negative Sine Value (medium)

Solve $\sin x = -0.45$ on $[0, 2\pi)$. Give answers to four decimal places.

Problem 14 Tangent Equation with Technology (medium)

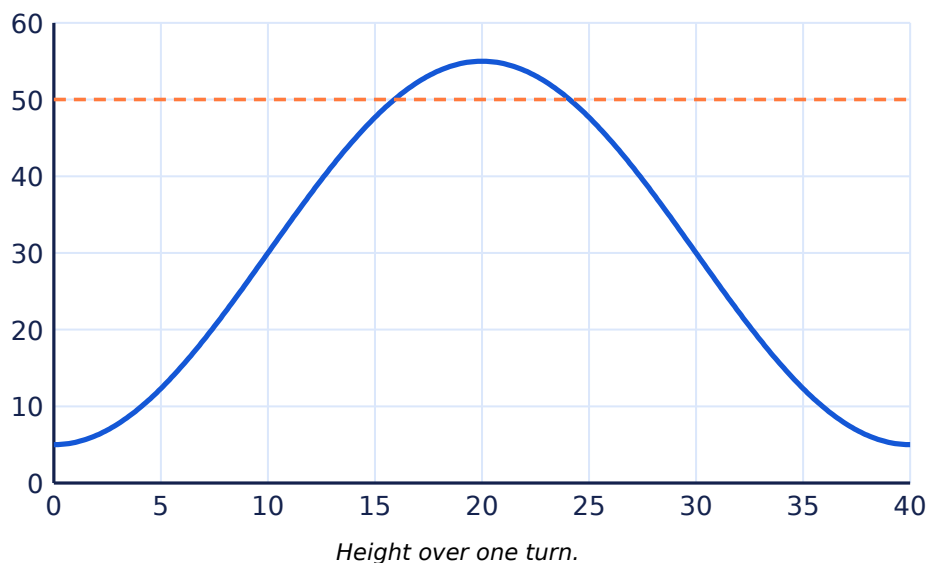
Solve $3 \tan x + 5 = 0$ on $[0, 2\pi)$. Give answers to four decimal places.

Problem 15 Factor Instead of Dividing (hard)

Solve $2 \sin x \cos x = \cos x$ on $[0, 2\pi)$. Explain why dividing both sides by $\cos x$ would lose solutions.

Problem 16 Ferris Wheel at 50 Feet (hard)

A rider's height is $H(t) = 30 - 25\cos(\pi t/20)$ feet, t in seconds, for one turn $0 \leq t \leq 40$. When is the rider 50 feet above the ground? Round to the nearest tenth of a second.

**Problem 17 Tide Window** (hard)

Water depth at a dock is $h(t) = 3.5 \sin(\pi t/6) + 4.5$ feet, where t is the hours after midnight ($0 \leq t \leq 12$). A boat needs at least 6.5 feet of water. During which times is the depth at least 6.5 feet?

Problem 18 Wheelchair Ramp Angles (hard)

A ramp 14 feet long rises 1.1 feet. (a) Find its angle of elevation using arcsin, in degrees. (b) The maximum allowed slope is 1 foot of rise per 12 feet of horizontal run. Find the corresponding angle with arctan. (c) How high can a 20-foot ramp rise at that maximum angle?

Problem 19 Swinging Pendulum (hard)

A pendulum's angle from vertical is $\theta(t) = 0.35\cos(5\pi t/4)$ radians, t in seconds. Find the first two times in one period when the angle is 0.2 radian, to three decimals.

Problem 20 Quadratic in Sine with an Impossible Factor (hard)

Solve $2 \sin^2 x + 3 \sin x - 2 = 0$ on $[0, 2\pi)$. Explain why one factor gives no solution, then write the general solutions.