

Law of Sines, Law of Cosines and Area

Grade 12 – Precalculus math · Lesson · Standards: G-SRT.D.9, G-SRT.D.10, G-SRT.D.11

Quick answer: The Law of Sines, $a/\sin A = b/\sin B = c/\sin C$, solves triangles given ASA, AAS or SSA, and the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$, solves SAS and SSS. Triangle area is $\frac{1}{2}ab \sin C$ or Heron's formula, and the SSA case can produce zero, one or two triangles.

Standards: G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · **Level:** Grade 12 Precalculus

How do you find the area of a triangle from two sides and an angle?

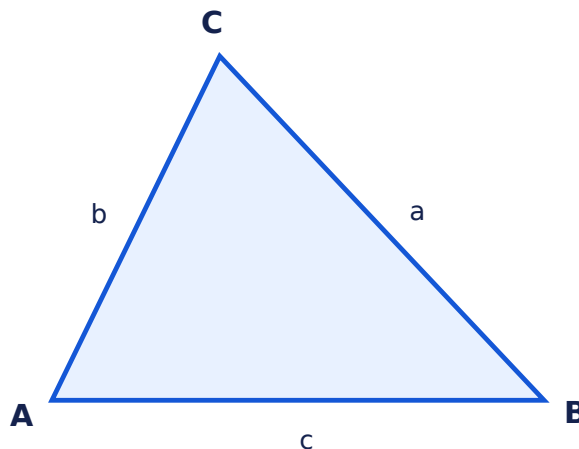
If sides a and b enclose angle C , the height to side a is $b \sin C$, so the area is $\frac{1}{2}ab \sin C$. When all three sides are known, use Heron's formula.

Area formulas

- Area = $\frac{1}{2}ab \sin C$ (two sides and the included angle).
- Heron: with $s = (a + b + c)/2$, Area = $\sqrt{s(s - a)(s - b)(s - c)}$.

Example: sides 9, 10 and 17 give $s = 18$ and area = $\sqrt{(18 \cdot 9 \cdot 8 \cdot 1)} = \sqrt{1296} = 36$.

When do you use the Law of Sines and the Law of Cosines?



Standard labeling: side a is opposite angle A , and so on.

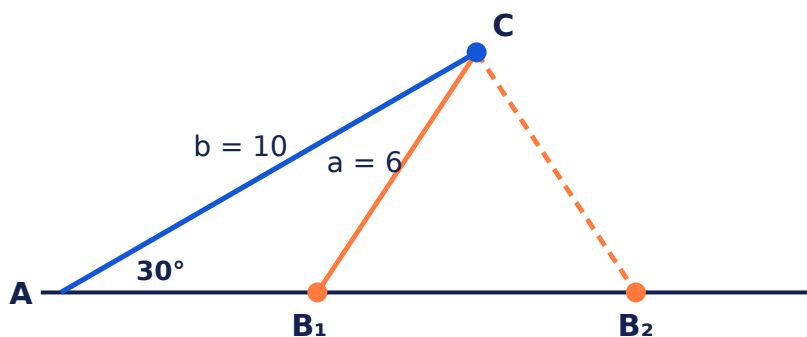
Given	Use	Formula
ASA, AAS, or SSA	Law of Sines	$a/\sin A = b/\sin B = c/\sin C$
SAS or SSS	Law of Cosines	$c^2 = a^2 + b^2 - 2ab \cos C$

Worked example: Law of Cosines

With $a = 8$, $b = 11$ and $C = 52^\circ$: $c^2 = 64 + 121 - 176 \cos 52^\circ \approx 76.64$, so $c \approx 8.75$.

What is the ambiguous case of the Law of Sines?

Given angle A and sides a and b (SSA), compare a with the height $h = b \sin A$. If $a < h$, no triangle exists. If $a = h$, one right triangle. If $h < a < b$, two triangles. If $a \geq b$, one triangle.

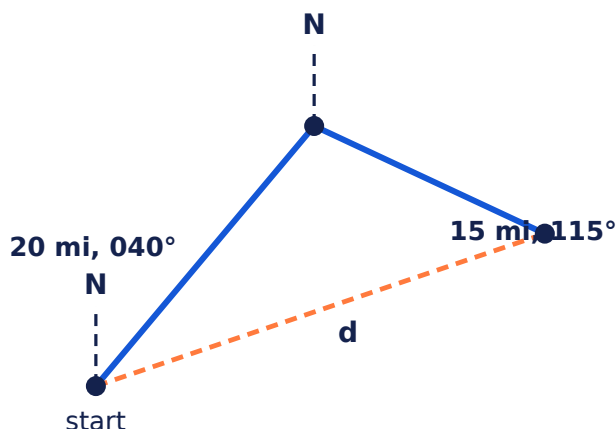


Angle $A = 30^\circ$, $b = 10$ and $a = 6$ fit two triangles.

In the figure, $h = 10 \sin 30^\circ = 5$ and $5 < 6 < 10$, so two triangles fit. The second angle is $B_2 = 180^\circ - B_1$.

How do bearings work in navigation problems?

A bearing is the angle measured clockwise from north. Draw a north line at every turning point, find the interior angle of the triangle from the bearings, then apply the Law of Cosines or Sines.



Bearings are measured clockwise from north.

Worked example: ship path

The interior angle at the turn is $180^\circ - (115^\circ - 40^\circ) = 105^\circ$. Then $d^2 = 20^2 + 15^2 - 2(20)(15)\cos 105^\circ \approx 780.29$, so $d \approx \mathbf{27.9 \text{ miles}}$.

Common mistake

Using the Law of Sines for SAS. You need a side and its opposite angle for a pair, and SAS does not give one. Use the Law of Cosines first.

Mathbot's tip

Make a sketch and label the sides a, b, c opposite angles A, B, C . Check that the longest side faces the largest angle.

Key points to remember

- Area = $\frac{1}{2}ab \sin C$; Heron: $\sqrt{s(s-a)(s-b)(s-c)}$.
- Law of Sines for ASA, AAS, SSA; Law of Cosines for SAS, SSS.
- SSA can give no triangle, one right triangle, one triangle, or two triangles.
- A bearing is measured clockwise from north.
- The largest angle is opposite the longest side.

Law of Sines, Law of Cosines and Area: frequently asked questions

When should you use the Law of Sines instead of the Law of Cosines?

Use the Law of Sines when you know a side and its opposite angle (ASA, AAS, SSA). Use the Law of Cosines when you know two sides and the included angle (SAS) or all three sides (SSS).

What is the ambiguous case in the Law of Sines?

With two sides and a non-included angle (SSA), the data can describe zero, one or two triangles. Compare the side opposite the angle with $b \sin A$, and if two triangles fit, use B and $180^\circ - B$.

How does Heron's formula find the area?

It uses only the three side lengths. Compute the semiperimeter $s = (a + b + c)/2$, then take the square root of $s(s-a)(s-b)(s-c)$. No angle or height is needed.

Why does the area equal $\frac{1}{2}ab \sin C$?

Taking side a as the base, the height is $b \sin C$ because the height is the vertical leg of a right triangle with hypotenuse b and angle C . Then area = $\frac{1}{2} \times \text{base} \times \text{height}$.

What is a bearing?

A bearing is a direction given as an angle measured clockwise from north, usually written with three digits. A bearing of 090° is east, 180° is south and 270° is west.