

Law of Sines, Law of Cosines and Area

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: G-SRT.D.9, G-SRT.D.10, G-SRT.D.11

Each answer names the law used, shows the substitution and ends with the result in bold. Draw and label the triangle first, and check that the longest side faces the largest angle.

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Problem 1 Area from Two Sides and an Angle

$$\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2}(14)(9) \sin 38^\circ = 63 \times 0.6157 \approx \mathbf{38.8 \text{ square inches.}}$$

Problem 2 Heron's Formula

$$s = (20 + 37 + 51)/2 = 54. \text{ Area} = \sqrt{(54 \cdot 34 \cdot 17 \cdot 3)} = \sqrt{93,636} = \mathbf{306 \text{ square inches.}}$$

Problem 3 Law of Sines with AAS

$$C = 180^\circ - 48^\circ - 71^\circ = \mathbf{61^\circ}.$$

$$b = 12 \sin 71^\circ \div \sin 48^\circ \approx \mathbf{15.27}. \quad c = 12 \sin 61^\circ \div \sin 48^\circ \approx \mathbf{14.12}.$$

Problem 4 Law of Cosines for the Third Side

$$c^2 = 64 + 121 - 2(8)(11) \cos 52^\circ = 185 - 176(0.6157) \approx 76.64.$$

$$c \approx \mathbf{8.75}.$$

Problem 5 Third Side from a Figure

$$\text{Law of Cosines: } a^2 = 49 + 81 - 2(7)(9) \cos 64^\circ = 130 - 126(0.4384) \approx 74.77.$$

$$a \approx \mathbf{8.65}.$$

Problem 6 Angle from Three Sides

$$\text{The largest angle is opposite 15: } \cos C = (64 + 100 - 225)/(2 \cdot 8 \cdot 10) = -61/160 \approx -0.3813.$$

$$C \approx \mathbf{112.4^\circ}.$$

Problem 7 Law of Sines with ASA

$$B = 180^\circ - 36^\circ - 79^\circ = \mathbf{65^\circ}.$$

$$a = 20 \sin 36^\circ \div \sin 65^\circ \approx \mathbf{12.97}. \quad c = 20 \sin 79^\circ \div \sin 65^\circ \approx \mathbf{21.66}.$$

Problem 8 SSA with One Triangle

Since $a > b$, exactly one triangle exists. $\sin B = 11 \sin 50^\circ \div 15 \approx 0.5618$, so $B \approx \mathbf{34.2^\circ}$.

$$C = 180^\circ - 50^\circ - 34.2^\circ \approx \mathbf{95.8^\circ}, \text{ and } c = 15 \sin C \div \sin 50^\circ \approx \mathbf{19.5}.$$

Problem 9 SSA with No Triangle

The height is $h = b \sin A = 9 \sin 40^\circ \approx 5.79$. Since $a = 5$ is less than h , the side is too short to reach the base line.

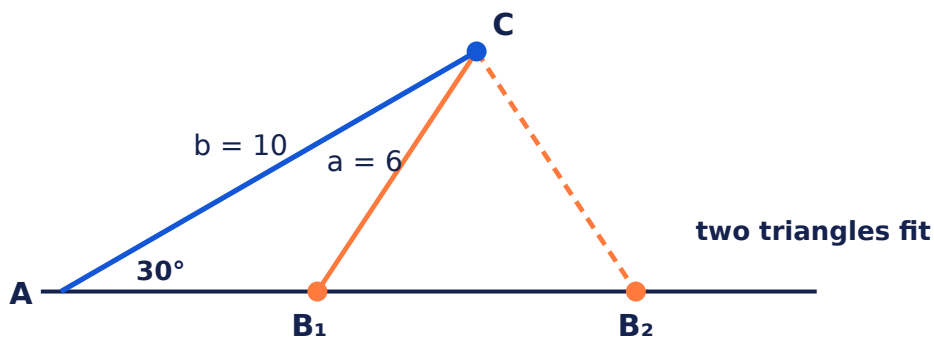
Equivalently $\sin B = 9 \sin 40^\circ \div 5 \approx 1.157 > 1$. **No triangle exists.**

Problem 10 SSA with Two Triangles

The height is $13 \sin 32^\circ \approx 6.89$, and $6.89 < 9 < 13$, so two triangles fit.

$\sin B = 13 \sin 32^\circ \div 9 \approx 0.7654$. Triangle 1: $B_1 \approx \mathbf{49.9^\circ}$, $C_1 \approx \mathbf{98.1^\circ}$, $c_1 = 9 \sin C_1 \div \sin 32^\circ \approx \mathbf{16.8}$.

Triangle 2: $B_2 = 180^\circ - 49.9^\circ \approx \mathbf{130.1^\circ}$, $C_2 \approx \mathbf{17.9^\circ}$, $c_2 \approx \mathbf{5.2}$.



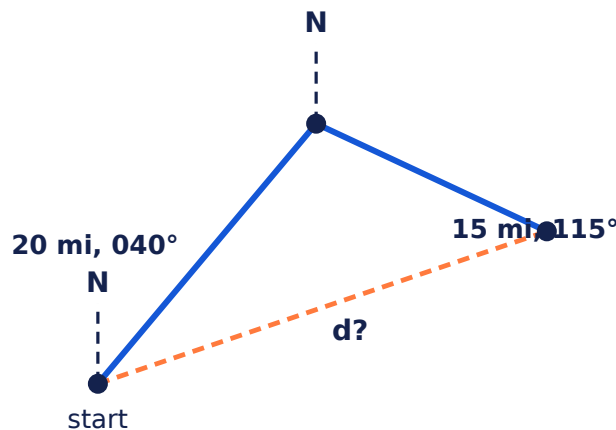
Angle $A = 30^\circ$, $b = 10$ and $a = 6$ fit two triangles.

Problem 11 Ship on Two Bearings

The interior angle at the turn is $180^\circ - (115^\circ - 40^\circ) = 105^\circ$.

$$d^2 = 20^2 + 15^2 - 2(20)(15) \cos 105^\circ = 625 + 155.29 \approx 780.3.$$

$$d \approx \mathbf{27.9 \text{ miles.}}$$



Path of the ship.

Problem 12 Area from Three Decimal Sides

$s = (7.2 + 8.5 + 10.1)/2 = 12.9$. Area = $\sqrt{(12.9 \cdot 5.7 \cdot 4.4 \cdot 2.8)} = \sqrt{905.89} \approx \mathbf{30.1 \text{ square centimeters.}}$

Problem 13 Find the Area After Solving

$$b = 10 \sin 63^\circ \div \sin 52^\circ \approx \mathbf{11.3} \text{ and } C = 180^\circ - 52^\circ - 63^\circ = \mathbf{65^\circ}.$$

$$\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2}(10)(11.31) \sin 65^\circ \approx \mathbf{51.2 \text{ square units.}}$$

Problem 14 Area of a Parallelogram

A diagonal splits the parallelogram into two triangles with equal area $\frac{1}{2}(12)(18) \sin 65^\circ$. The parallelogram is twice that: $12 \cdot 18 \cdot \sin 65^\circ = 216 \times 0.9063 \approx \mathbf{195.8 \text{ square units.}}$

Problem 15 All Three Angles from Three Sides

Largest angle (opposite 20): $\cos = (144 + 225 - 400)/(2 \cdot 12 \cdot 15) = -31/360$, so $\mathbf{94.9^\circ}$.

Opposite 15: $\cos = (144 + 400 - 225)/(2 \cdot 12 \cdot 20) = 319/480$, so $\mathbf{48.3^\circ}$.

The last angle is $180^\circ - 94.9^\circ - 48.3^\circ \approx \mathbf{36.7^\circ}$ (opposite 12).

Problem 16 Distance Across a Pond

$$AB^2 = 210^2 + 165^2 - 2(210)(165) \cos 74^\circ = 71,325 - 69,300(0.2756) \approx 52,223.$$

$$AB \approx \mathbf{228.5 \text{ feet.}}$$

Problem 17 Locating a Fire from Two Stations

The angle at the fire is $180^\circ - 62^\circ - 48^\circ = 70^\circ$.

Law of Sines: distance from A = $8.4 \sin 48^\circ \div \sin 70^\circ \approx \mathbf{6.64 \text{ miles}}$; distance from B = $8.4 \sin 62^\circ \div \sin 70^\circ \approx \mathbf{7.89 \text{ miles}}$.

Problem 18 Hiker Back to Camp

The interior angle at the turn is $180^\circ - (140^\circ - 25^\circ) = 65^\circ$.

$$d^2 = 3.5^2 + 2.2^2 - 2(3.5)(2.2) \cos 65^\circ \approx 10.58, \text{ so } d \approx \mathbf{3.3 \text{ miles.}}$$

$$\sin(\text{angle at camp}) = 2.2 \sin 65^\circ \div 3.253 \approx 0.6129, \text{ so the angle is } \approx \mathbf{37.8^\circ}.$$

Problem 19 Triangular Lot

$$s = (120 + 150 + 190)/2 = 230. \text{ Area} = \sqrt{(230 \cdot 110 \cdot 80 \cdot 40)} = \sqrt{80,960,000} \approx \mathbf{8,998 \text{ square feet.}}$$

$$\text{Cost} \approx 8,998 \times 0.85 \approx \mathbf{\$7,648}.$$

Problem 20 A Garden with Two Possible Shapes

The height is $18 \sin 28^\circ \approx 8.45$ and $8.45 < 11 < 18$, so two shapes fit.

$\sin B = 18 \sin 28^\circ \div 11 \approx 0.7682$. Shape 1: $B \approx 50.2^\circ$, $C \approx 101.8^\circ$, $AB = 11 \sin C \div \sin 28^\circ \approx \mathbf{22.9 \text{ ft}}$, area = $\frac{1}{2}(18)(22.9) \sin 28^\circ \approx \mathbf{96.9 \text{ ft}^2}$.

Shape 2: $B \approx 129.8^\circ$, $C \approx 22.2^\circ$, $AB \approx \mathbf{8.9 \text{ ft}}$, area $\approx \mathbf{37.4 \text{ ft}^2}$.