

Exponents, Logarithms and Growth Models Explained

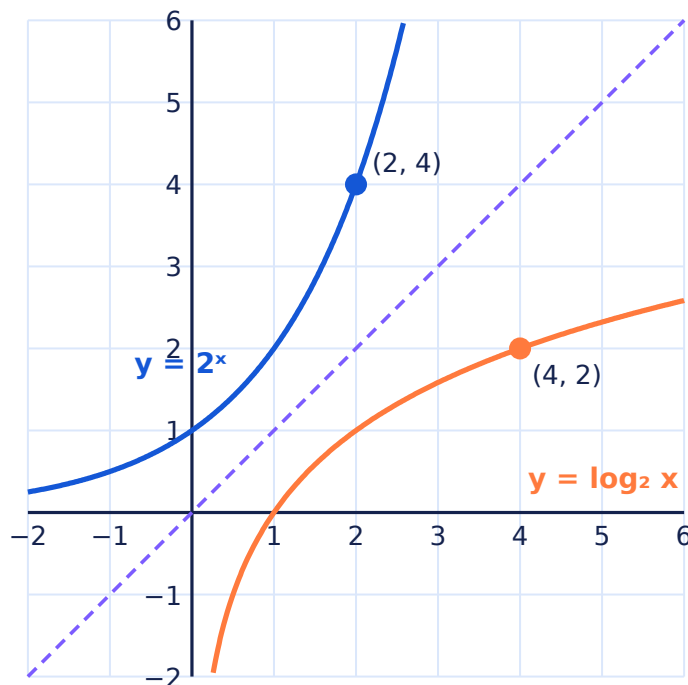
Grade 12 – Precalculus math · Lesson · Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b

Quick answer: A logarithm is the exponent that produces a number: log base b of x equals y exactly when b to the y equals x . Exponentials and logarithms are inverse functions, and the laws of logarithms let you solve growth equations, logistic models and logarithmic scales such as pH and magnitude.

Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b · **Level:** Grade 12 Precalculus

How are exponents and logarithms related?

A logarithm answers the question: what exponent is needed? The statement $\log_b x = y$ means exactly that $b^y = x$, with base $b > 0$, $b \neq 1$, and $x > 0$. Because the two functions undo each other, $\log_b(b^x) = x$ and $b^{\log_b x} = x$. Their graphs mirror each other across $y = x$. The natural logarithm $\ln$ uses base $e \approx 2.718$.



An exponential and its logarithm are reflections across $y = x$.

What are the laws of logarithms?

Law	Statement
Product	$\log_b(MN) = \log_b M + \log_b N$
Quotient	$\log_b(M/N) = \log_b M - \log_b N$
Power	$\log_b(M^k) = k \cdot \log_b M$
Change of base	$\log_b M = \ln M \div \ln b$

How do you solve exponential and logarithmic equations?

Method: isolate, then undo

1. Isolate the exponential or the logarithm.
2. Take a logarithm of both sides (exponential equation) or rewrite in exponential form (log equation).
3. Solve, then check every answer in the original equation.

Worked example: exponential

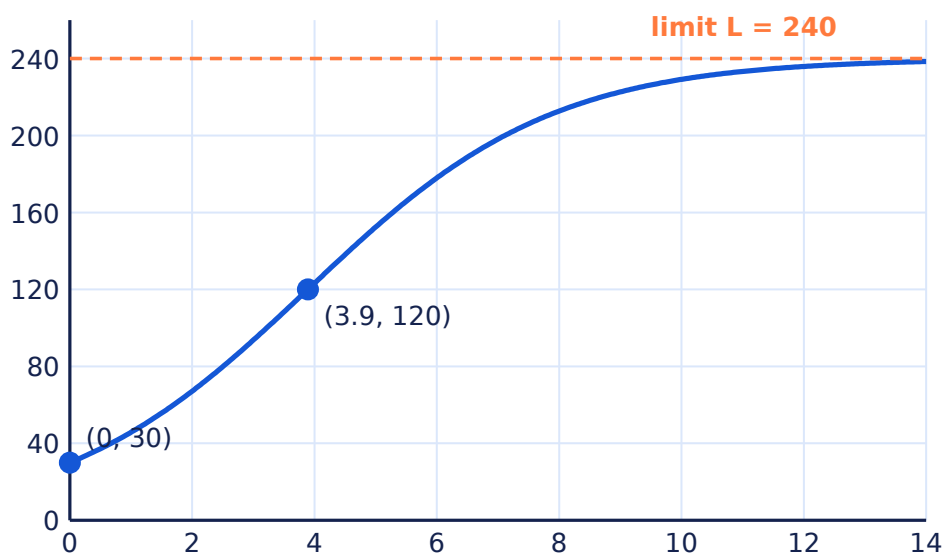
$5^{2x-1} = 40$ gives $(2x - 1) \ln 5 = \ln 40$, so $2x - 1 = \ln 40 \div \ln 5 \approx 2.2920$ and $x \approx \mathbf{1.646}$.

Worked example: logarithmic

$\log_3(x + 2) + \log_3(x - 4) = 3$ becomes $(x + 2)(x - 4) = 27$, so $x^2 - 2x - 35 = 0$ and $x = 7$ or $x = -5$. The value -5 makes a logarithm undefined, so the only solution is $x = \mathbf{7}$.

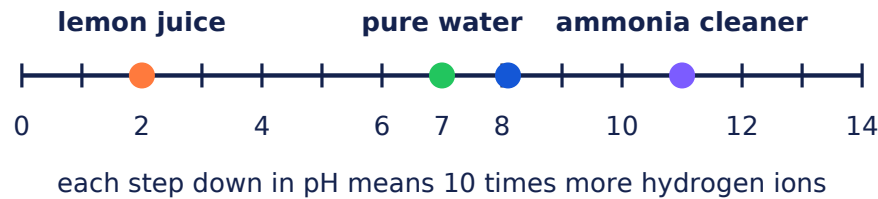
How do growth models and logarithmic scales work?

Continuous growth uses $A = Pe^{rt}$. Logistic growth, $N(t) = L \div (1 + Ae^{-kt})$, starts almost exponentially but levels off at the carrying capacity L .



Logistic growth levels off at the carrying capacity L .

Logarithmic scales compress huge ranges. Each whole step in pH, decibel level $\div 10$, or earthquake magnitude multiplies the measured quantity by a fixed factor.



The pH scale is logarithmic: one unit means a factor of 10.

Common mistake

Writing $\log_b(M + N) = \log_b M + \log_b N$. The product law needs MN inside, not a sum.

Mathbot's tip

After solving a log equation, plug each answer back in. A negative or zero input to a logarithm means that answer must be thrown out.

Key points to remember

- $\log_b x = y$ means $b^y = x$; the inputs of a logarithm must be positive.
- Product, quotient and power laws turn multiplication into addition, division into subtraction and exponents into factors.
- Change of base: $\log_b M = \ln M \div \ln b$.
- Check every solution of a log equation, because extraneous answers are common.
- Logistic growth levels off at a carrying capacity; continuous growth is $A = Pe^{(rt)}$.

Exponents, Logarithms and Growth Models Explained: frequently asked questions

What is a logarithm in simple terms?

It is the exponent you must put on a base to get a given number. Since 2 to the 5th power is 32, the logarithm base 2 of 32 equals 5.

How do you solve an exponential equation when the bases do not match?

Isolate the power, then take the natural log of both sides and use the power law to bring the exponent down. Divide to finish. For $7^x = 19$, $x = \ln 19 \div \ln 7$.

Why do logarithmic equations give extraneous solutions?

Combining logs can produce a quadratic whose roots include values that make an original logarithm have a zero or negative input. Always substitute each root back into the original equation.

How is logistic growth different from exponential growth?

Exponential growth never stops increasing. Logistic growth speeds up at first, then slows and approaches a carrying capacity, which is the horizontal asymptote of the curve.

Why are pH and earthquake magnitude logarithmic scales?

The quantities they measure vary over many powers of ten. Using the logarithm compresses them into small numbers, so one unit of pH or magnitude means a factor of 10.