

# Exponents, Logarithms and Growth Models Explained

**Grade 12 – Precalculus math · Test Answer Key · Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b**

Each solution gives the exact and decimal answers with a scoring note. Award method points separately from the final decimal so a rounding slip costs at most one point.

**Standards:** F-LE.A.4, F-BF.B.5, F-IF.C.8b · **Level:** Grade 12 Precalculus

## Question 1 Evaluate Logarithms (2 points)

a.  $3^{-4} = 1/81$ , so **-4**.

b.  $8^{1/3} = 2$ , so **1/3**.

*Scoring: 1 point per value.*

## Question 2 Expand and Condense (2 points)

(a)  $\log_5 25 + 2 \log_5 x - \log_5 y^{1/2} = 2 + 2 \log_5 x - (1/2) \log_5 y$ .

(b)  $\ln 9 + \ln 16 - \ln 6 = \ln(144/6) = \ln 24$ .

*Scoring: 1 point per part.*

## Question 3 Solve an Exponential Equation (2 points)

$(2x - 1) \ln 3 = \ln 14$ , so  $2x - 1 = \ln 14 \div \ln 3 \approx 2.4022$ .

$x = (1 + \ln 14 \div \ln 3)/2 \approx \mathbf{1.701}$ .

*Scoring: 1 point for taking logs correctly; 1 point for the exact and decimal answer.*

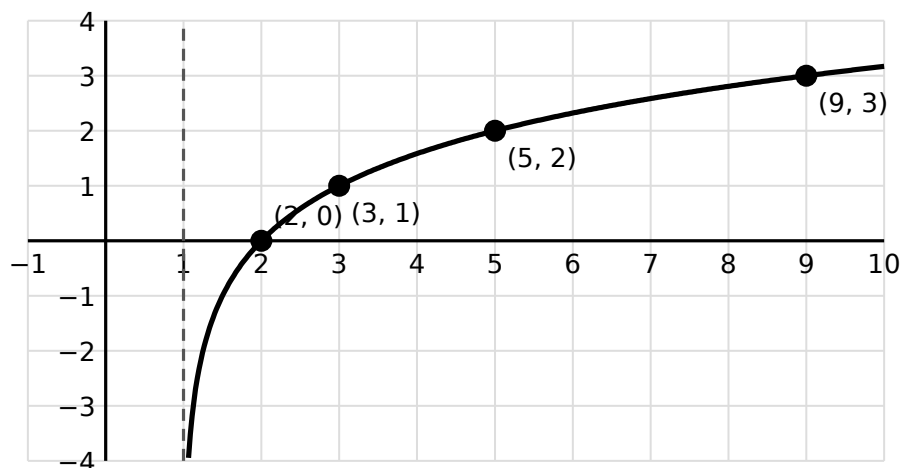
## Question 4 Logarithmic Equation (3 points)

Combine:  $\log_6[x(x - 5)] = 1$ , so  $x^2 - 5x = 6$  and  $x^2 - 5x - 6 = (x - 6)(x + 1) = 0$ .

The candidate  $-1$  makes  $\log_6(-1)$  undefined. The candidate  $6$  gives  $\log_6 6 + \log_6 1 = 1 + 0 = 1$   
✓.  **$x = 6$** .

*Scoring: 1 point for combining logs; 1 point for solving the quadratic; 1 point for rejecting  $-1$ .*

### Question 5 Graph of a Shifted Logarithm (2 points)



The point  $(9, 3)$  answers part (b).

- a. Asymptote  $x = 1$ ; domain  $(1, \infty)$ .
- b. Rewrite  $\log_2(x - 1) = 3$  as  $x - 1 = 8$ , so  $x = 9$ .

Scoring: 1 point per part.

### Question 6 Continuous Growth of a Town (3 points)

- a.  $P(10) = 18500e^{0.24} \approx \mathbf{23,518}$  people.
- b. Solve  $e^{0.024t} = 2$ :  $t = \ln 2 \div 0.024 \approx \mathbf{28.9}$  years.
- c.  $e^{0.024t} = 30000/18500$ , so  $t = \ln(1.6216) \div 0.024 \approx \mathbf{20.1}$  years.

Scoring: 1 point per part.

### Question 7 Logistic Model of a Herd (2 points)

- a. Capacity **900**;  $N(0) = 900 \div 18 = \mathbf{50}$ .
- b.  $1 + 17e^{-0.6t} = 2$ , so  $e^{-0.6t} = 1/17$  and  $t = \ln 17 \div 0.6 \approx \mathbf{4.72}$  years.

Scoring: 1 point per part.

### Question 8 Logarithmic Scales (3 points)

- (a)  $7.1 - 5.4 = 1.7$ , so the ratio is  $10^{1.7} \approx \mathbf{50.1}$  times.
- (b)  $M = \log 2000 \approx \mathbf{3.30}$ .
- (c)  $[H^+] = 10^{-3.4} \approx \mathbf{3.98 \times 10^{-4}}$  mol/L.

Scoring: 1 point per part.

**Question 9 Find an Inverse (1 point)**

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Swap and solve:  $x = \ln(y - 3) + 1$ , so  $\ln(y - 3) = x - 1$  and  $y = e^{x-1} + 3$ .

**$f^{-1}(x) = e^{x-1} + 3$ .**

*Scoring: 1 point for the correct inverse.*