

Exponents, Logarithms and Growth Models Explained

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b

Each answer shows the logarithm or exponent step, the exact form where it helps, and a decimal in bold. Check every logarithmic solution in the original equation before accepting it.

Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b · **Level:** Grade 12 Precalculus

Problem 1 Switch Between Forms

(a) $\log_6 216 = 3$.

(b) $9^2 = 81$.

(c) $\log 0.01 = -2$ (base 10).

Problem 2 Evaluate Logarithms Exactly

(a) $2^6 = 64$, so **6**. (b) $5^{-2} = 1/25$, so **-2**. (c) $9^{1/2} = 3$, so **1/2**. (d) The natural log undoes e^4 : **4**.

Problem 3 Expand a Logarithm

$$\log_3(27a^2) - \log_3 b = \log_3 27 + \log_3 a^2 - \log_3 b.$$

$$3 + 2 \log_3 a - \log_3 b.$$

Problem 4 Condense a Logarithmic Expression

$$2 \ln 5 = \ln 25. \text{ Then } \ln 25 - \ln 10 = \ln 2.5 \text{ and } \ln 2.5 + \ln 4 = \ln 10.$$

$$\ln 10.$$

Problem 5 Match the Bases

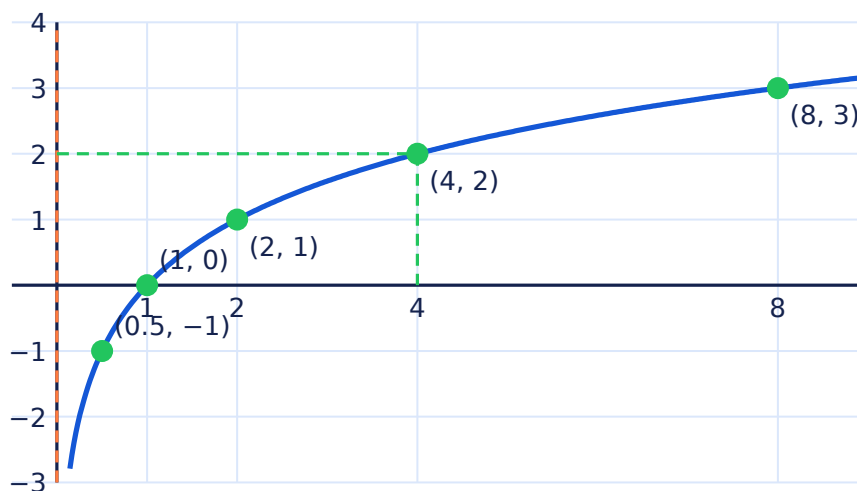
(a) $32 = 2^5$, so **x = 5**.

(b) $81 = 3^4$, so $x + 1 = 4$ and **x = 3**.

Problem 6 Read a Logarithmic Graph

The point (4, 2) gives $\log_2 4 = 2$. The point (8, 3) gives $\log_2 x = 3$ at $x = 8$.

The curve exists only for positive x : domain $(0, \infty)$, vertical asymptote $x = 0$.



Reading $\log_2 4 = 2$ from the point (4, 2).

Problem 7 Change of Base

$$\log_7 50 = \ln 50 \div \ln 7 \approx 3.9120 \div 1.9459 \approx \mathbf{2.010}.$$

$$\log_4 0.3 = \ln 0.3 \div \ln 4 \approx -1.2040 \div 1.3863 \approx \mathbf{-0.868}.$$

Problem 8 Exponential Equation with Logs

Take $\ln$ of both sides: $(x - 2) \ln 7 = \ln 19$, so $x - 2 = \frac{(\ln 19)}{(\ln 7)} \approx 1.5131$.

$$\mathbf{x = 2 + \frac{(\ln 19)}{(\ln 7)} \approx 3.513}.$$

Problem 9 Isolate a Natural Exponential

Divide by 3: $e^{0.4x} = 7$. Take $\ln$: $0.4x = \ln 7$.

$$\mathbf{x = \frac{(\ln 7)}{0.4} \approx 4.865}.$$

Problem 10 Logarithmic Equation with a Check

Combine: $\log_3[(x + 1)(x - 5)] = 3$, so $x^2 - 4x - 5 = 27$ and $x^2 - 4x - 32 = (x - 8)(x + 4) = 0$.

For $x = -4$ the inputs -3 and -9 are negative: rejected. For $x = 8$: $\log_3 9 + \log_3 3 = 2 + 1 = 3$

✓. $x = 8$.

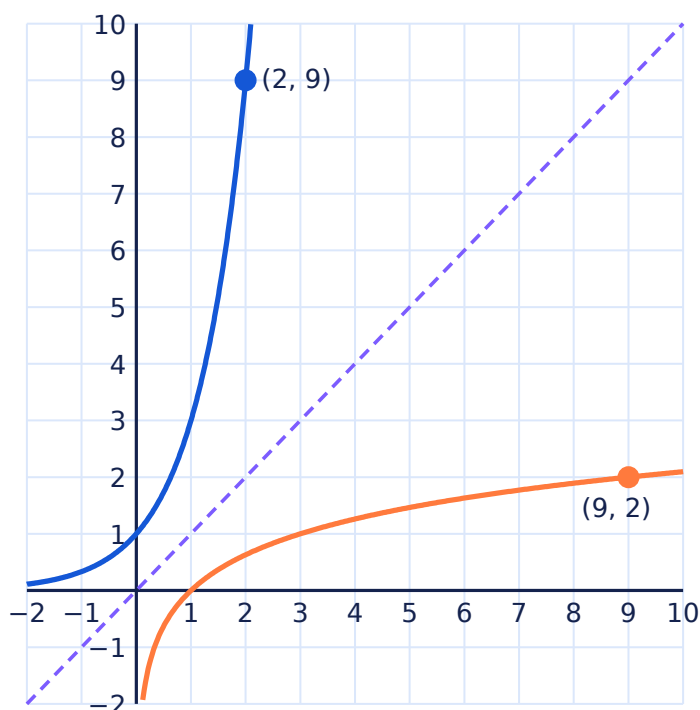
Problem 11 Natural Log Equation

Rewrite as $2x - 1 = e^3$. Then $x = (e^3 + 1)/2$.

$x = (e^3 + 1)/2 \approx \mathbf{10.543}$.

Problem 12 Inverse Pairs and Reflections

Swapping coordinates gives the point (9, 2) on g . Since $3^3 = 27$, $g(27) = \mathbf{3}$. The range of g is the domain of f : **all real numbers**.



The point (2, 9) reflects to (9, 2).

Problem 13 Solve with a Fractional Exponent

Divide by 6: $2^{x/3} = 16 = 2^4$. So $x/3 = 4$.

$x = \mathbf{12}$.

Problem 14 Doubling Culture

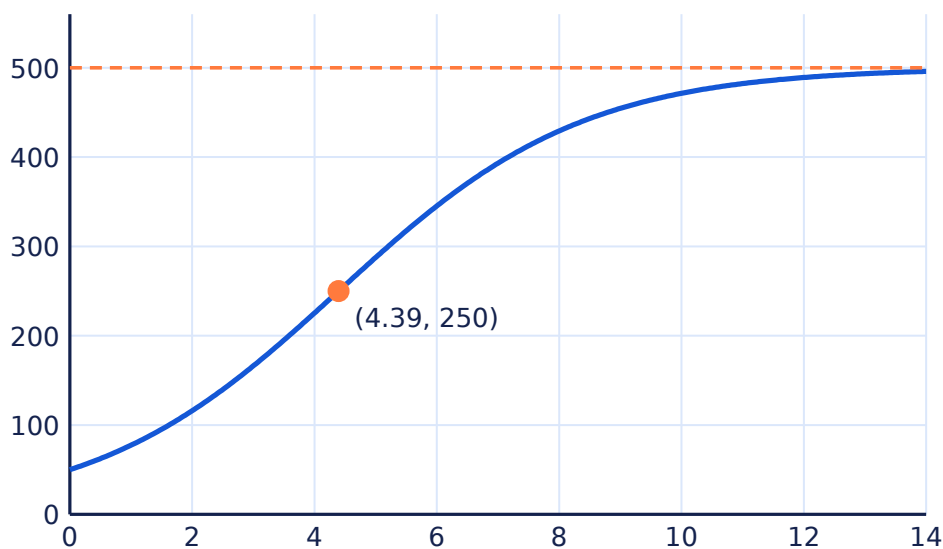
8,000 cells: $2^{t/3} = 16$, so $t/3 = 4$ and $t = \mathbf{12 \text{ hours}}$ (the marked point).

20,000 cells: $2^{t/3} = 40$, so $t = 3 \cdot (\ln 40) / (\ln 2) \approx \mathbf{15.97 \text{ hours}}$.

Problem 15 Logistic Starting Value and Limit

$L(0) = 500 \div (1 + 9) = \mathbf{50}$. As t grows, $e^{-0.5t} \rightarrow 0$, so the capacity is **500**.

Set $500 \div (1 + 9e^{-0.5t}) = 250$: $1 + 9e^{-0.5t} = 2$, so $e^{-0.5t} = 1/9$ and $t = 2 \ln 9 \approx \mathbf{4.39 \text{ years}}$.



Half of the capacity is reached at $t \approx 4.39$.

Problem 16 Continuously Compounded Savings

After 8 years: $2500e^{0.336} \approx \mathbf{\$3,498.35}$.

For \$4,000: $e^{0.042t} = 1.6$, so $t = \frac{(\ln 1.6)}{0.042} \approx \mathbf{11.19 \text{ years}}$.

Problem 17 Medicine Half-Life

25 mg: $(1/2)^{t/6} = 1/8 = (1/2)^3$, so $t/6 = 3$ and $t = \mathbf{18 \text{ hours}}$.

60 mg: $(1/2)^{t/6} = 0.3$, so $t = 6 \cdot \frac{(\ln 0.3)}{(\ln 0.5)} \approx \mathbf{10.42 \text{ hours}}$.

Problem 18 Comparing Earthquakes

The difference is $6.8 - 5.3 = 1.5$, so the amplitude ratio is $10^{1.5} \approx \mathbf{31.6 \text{ times}}$.

$M = \log 5000 \approx \mathbf{3.70}$.

Problem 19 Sound Levels

The difference is 45 dB, so $10 \log(\text{ratio}) = 45$ and the ratio is $10^{4.5} \approx \mathbf{31,623 \text{ times}}$.

Two sources double the intensity: $10 \log 2 \approx \mathbf{3.01 \text{ dB}}$ more.

Problem 20 Flu Spread in a School

400: $1 + 49e^{-0.35t} = 2$, so $e^{-0.35t} = 1/49$ and $t = (\ln 49)/_{0.35} \approx \mathbf{11.1 \text{ days}}$.

720: $1 + 49e^{-0.35t} = 10/9$, so $e^{-0.35t} = 1/441$ and $t = (\ln 441)/_{0.35} \approx \mathbf{17.4 \text{ days}}$.