

Exponents, Logarithms and Growth Models Explained

Grade 12 – Precalculus math · Worksheet · Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b

Name: _____ Date: _____ Class: _____

These 20 problems move from converting between exponential and logarithmic form to the laws of logarithms, solving equations, and modeling with logistic growth, half-life and logarithmic scales. They train standards F-LE.A.4, F-BF.B.5 and F-IF.C.8b. The last six problems use savings, medicine, earthquakes, sound and a flu outbreak.

Standards: F-LE.A.4, F-BF.B.5, F-IF.C.8b · **Level:** Grade 12 Precalculus

Problem 1 Switch Between Forms (easy)

Rewrite each statement in the other form. (a) $6^3 = 216$ (b) $\log_9 81 = 2$ (c) $10^{-2} = 0.01$

Problem 2 Evaluate Logarithms Exactly (easy)

Evaluate each without a calculator. (a) $\log_2 64$ (b) $\log_5(1/25)$ (c) $\log_9 3$ (d) $\ln e^4$

Problem 3 Expand a Logarithm (easy)

Expand $\log_3 \frac{27a^2}{b}$ using the laws of logarithms.

Problem 4 Condense a Logarithmic Expression (easy)

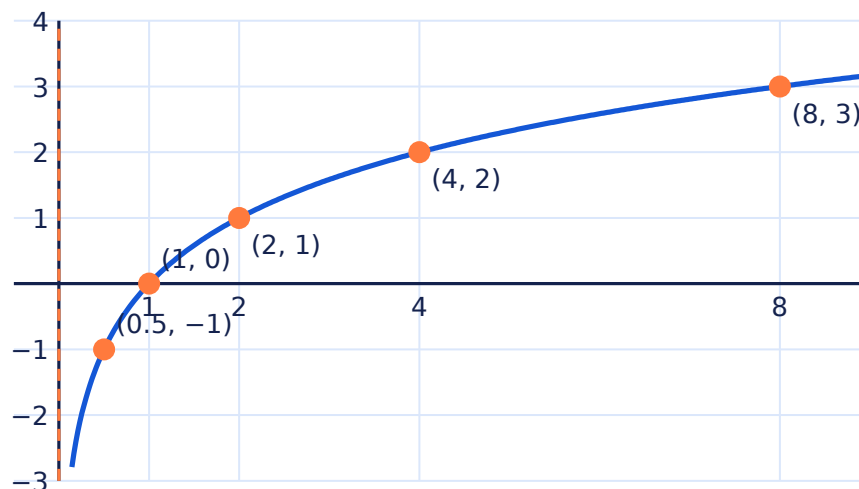
Write $2 \ln 5 - \ln 10 + \ln 4$ as a single logarithm and simplify.

Problem 5 Match the Bases (easy)

Solve each equation. (a) $2^x = 32$ (b) $3^{x+1} = 81$

Problem 6 Read a Logarithmic Graph (easy)

The graph shows $y = \log_2 x$. Use it to find $\log_2 4$, solve $\log_2 x = 3$, and state the domain and the vertical asymptote.



The graph of $y = \log_2 x$.

Problem 7 Change of Base (medium)

Use the change-of-base formula to approximate $\log_7 50$ and $\log_4 0.3$ to three decimal places.

Problem 8 Exponential Equation with Logs (medium)

Solve $7^{x-2} = 19$. Give the exact answer and a decimal to three places.

Problem 9 Isolate a Natural Exponential (medium)

Solve $3e^{0.4x} = 21$.

Problem 10 Logarithmic Equation with a Check (medium)

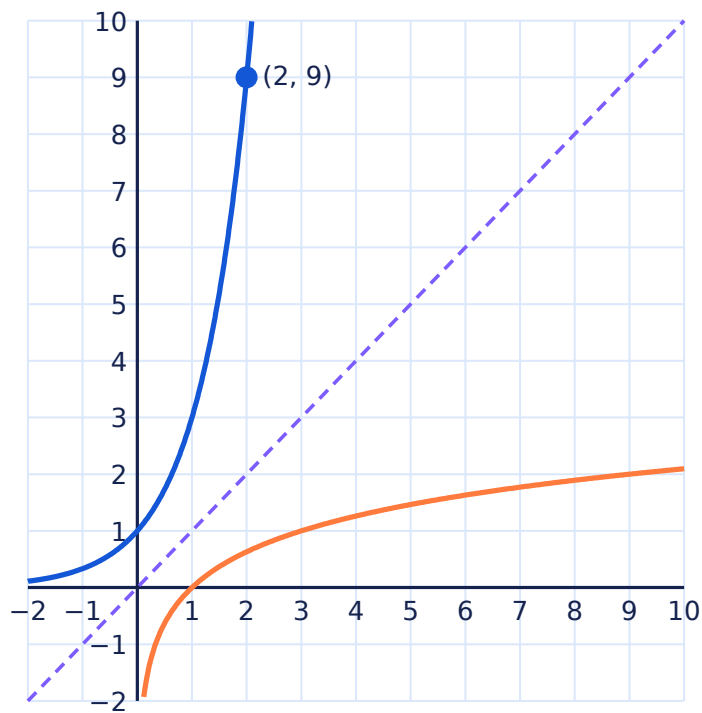
Solve $\log_3(x + 1) + \log_3(x - 5) = 3$ and check each candidate.

Problem 11 Natural Log Equation (medium)

Solve $\ln(2x - 1) = 3$. Give an exact answer and a decimal to three places.

Problem 12 Inverse Pairs and Reflections (medium)

The graph shows $f(x) = 3^x$ and its inverse $g(x) = \log_3 x$. Given that $f(2) = 9$, name a point on g , then find $g(27)$ and the range of g .



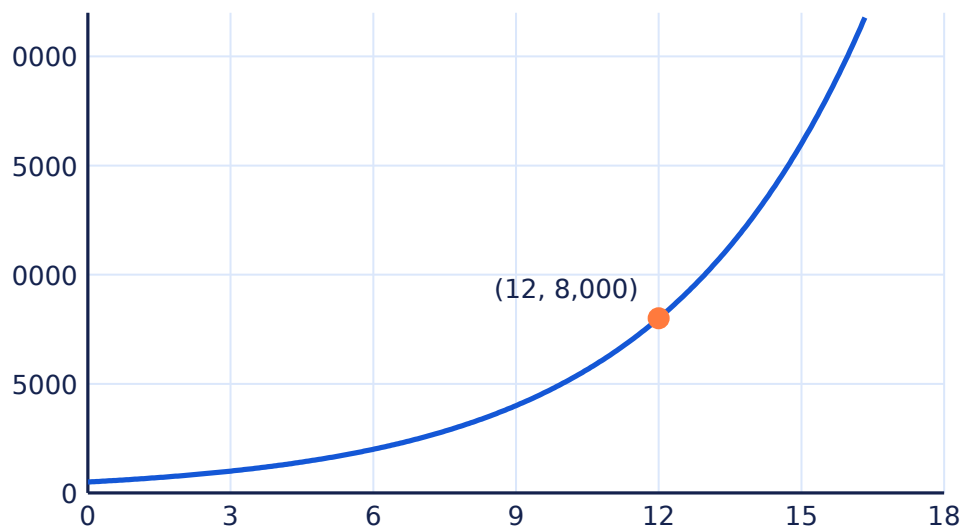
The graphs of f and g mirror each other.

Problem 13 Solve with a Fractional Exponent (medium)

Solve $6 \cdot 2^{x/3} = 96$.

Problem 14 Doubling Culture (medium)

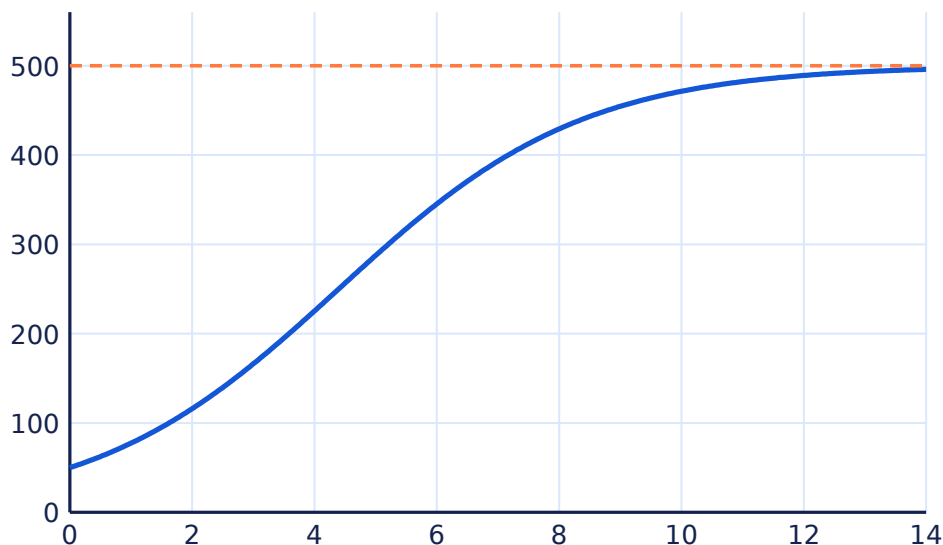
A culture starts with 500 cells and doubles every 3 hours, so $N(t) = 500 \cdot 2^{t/3}$. Find when the culture reaches 8,000 cells and when it reaches 20,000 cells.



Culture size over 18 hours.

Problem 15 Logistic Starting Value and Limit (medium)

A population follows $L(t) = \frac{500}{(1 + 9e^{-0.5t})}$ (t in years). Find the starting population, the carrying capacity, and the time when the population reaches 250.



Graph of $L(t)$.

Problem 16 Continuously Compounded Savings (hard)

A savings account holds \$2,500 and earns 4.2% interest compounded continuously, so $A = 2500e^{0.042t}$. Find the balance after 8 years and the time needed to reach \$4,000.

Problem 17 Medicine Half-Life (hard)

A 200 mg dose has a half-life of 6 hours, so $A(t) = 200(1/2)^{t/6}$. When are 25 mg left? When are 60 mg left?

Problem 18 Comparing Earthquakes (hard)

Magnitude is $M = \log(A/A_0)$, where A is the wave amplitude. How many times larger is the amplitude of a magnitude 6.8 quake than that of a magnitude 5.3 quake? What magnitude has an amplitude of $5,000A_0$?

Problem 19 Sound Levels (hard)

Sound level in decibels is $L = 10 \log(I/I_0)$. A concert measures 105 dB and a conversation 60 dB. How many times greater is the concert's intensity? By how many decibels does the level rise when two identical sources play together?

Problem 20 Flu Spread in a School (hard)

In a school of 800 students, the number infected after t days is $N(t) = \frac{800}{(1 + 49e^{-0.35t})}$. When are 400 students infected, and when do 720 (90%) become infected?