

Matrix Operations, Determinants and Inverses

Grade 12 – Precalculus math · Lesson · Standards: N-VM.C.6, N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10

Quick answer: A matrix arranges numbers in rows and columns. Equal-size matrices add position by position, and a product pairs each row of the first matrix with each column of the second. A square 2×2 matrix can be inverted only when $ad - bc$, its determinant, differs from zero.

Standards: N-VM.C.6, N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10 · **Level:** Grade 12 Precalculus

What is a matrix and how do you add or scale one?

A matrix organizes numbers into rows and columns. One with m rows and n columns is called $m \times n$, and each number inside is an entry. Two matrices of equal size are added or subtracted position by position, while multiplying by a scalar changes every entry. This ticket log uses rows for Friday and Saturday and columns for adult, child and senior tickets:

120	64	31
158	92	45

Identity and zero matrices

The zero matrix O holds only zeros, so adding it changes nothing: $A + O = A$. The identity matrix I carries 1s from its top-left corner down to its bottom-right corner, with zeros everywhere else. Multiplying by I on either side returns A unchanged.

How do you multiply two matrices?

You can multiply A by B only if the width of A matches the height of B . The answer is as tall as A and as wide as B . To fill the spot in row i and column j , add up the products of matching numbers taken from row i of A and column j of B .

$$\begin{array}{|c|c|c|} \hline \mathbf{A} & & \\ \hline 2 & -1 & 3 \\ \hline 0 & 4 & 1 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline \mathbf{B} & \\ \hline 1 & 2 \\ \hline 3 & 0 \\ \hline -2 & 5 \\ \hline \end{array} = \begin{array}{|c|c|} \hline \mathbf{AB} & \\ \hline -7 & 19 \\ \hline 10 & 5 \\ \hline \end{array}$$

$$\text{Entry (row 1, column 2)} = (2)(2) + (-1)(0) + (3)(5) = 19$$

The shaded row and column combine into the shaded entry of the product.

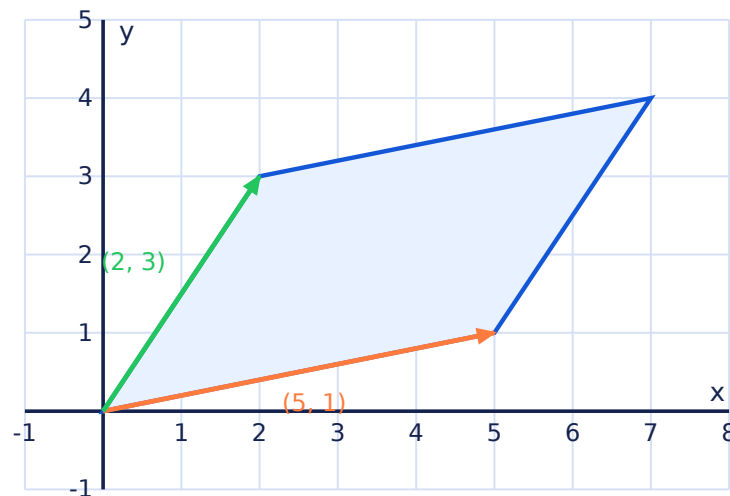
Common mistake

Multiplying two matrices position by position. The correct process pairs a row with a column, and swapping the factors usually changes the answer.

What is a determinant, and when does a 2×2 matrix have an inverse?

Determinant and inverse of a 2×2 matrix

For A with rows (a, b) and (c, d) , the determinant is $ad - bc$. A nonzero determinant gives $A^{-1} = \frac{1}{(ad - bc)}$ times the matrix with rows $(d, -b)$ and $(-c, a)$. A zero determinant means A is singular and cannot be inverted.



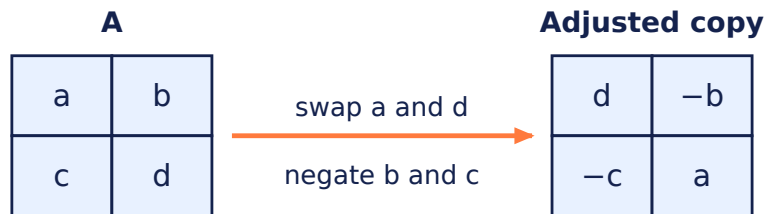
The area of this parallelogram is $|5 \cdot 3 - 2 \cdot 1| = 13$.

Method: finding an inverse

1. Calculate $ad - bc$ and stop if it equals 0.
2. Exchange the two diagonal entries and flip the sign of each off-diagonal entry.
3. Scale by dividing each entry by the determinant.
4. Verify that AA^{-1} gives I .

Worked example

Take A with rows $(4, -3)$ and $(2, 1)$. Its determinant is $4(1) - (-3)(2) = 10$. Exchanging and flipping signs yields rows $(1, 3)$ and $(-2, 4)$, and dividing by 10 produces A^{-1} with rows $(\frac{1}{10}, \frac{3}{10})$ and $(-\frac{1}{5}, \frac{2}{5})$.



then divide every entry by $ad - bc$

The three moves that turn A into A^{-1} .

Mathbot's tip

Multiply A by your inverse before you celebrate. Getting the identity matrix back is a thirty-second check that catches every sign slip.

Key points to remember

- An $m \times n$ matrix has m rows and n columns; sums need equal sizes.
- AB is defined only when the columns of A equal the rows of B, and $AB \neq BA$ in general.
- Identity I : $AI = IA = A$. Zero matrix O : $A + O = A$.
- det of the matrix with rows (a, b) and (c, d) is $ad - bc$.
- A^{-1} exists only if $\det A \neq 0$; the formula swaps a and d , negates b and c , and divides by $\det A$.

Matrix Operations, Determinants and Inverses: frequently asked questions

Why is matrix multiplication not commutative?

Each entry of AB comes from rows of A and columns of B, and swapping the order uses different rows and columns. Even when both AB and BA exist, the results usually differ, so the order of factors must be kept.

How can I tell if a product of matrices is defined?

Compare the inner sizes. An $m \times n$ matrix times a $p \times q$ matrix works only if $n = p$, and the product is $m \times q$.

What does a determinant of 0 mean?

The matrix is singular and has no inverse. Geometrically, it flattens the plane onto a line or a point, so the parallelogram built from its columns has zero area.

Does every square matrix have an inverse?

No. Only square matrices with a nonzero determinant do. For example, a matrix whose second row is twice its first row has determinant 0.

How do I check that two matrices are inverses?

Multiply them in either order. If the product is the identity matrix, they are inverses. If the product has any entry that differs from the identity, one of the matrices is wrong.