

Matrix Operations, Determinants and Inverses

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: N-VM.C.6, N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10

Each answer shows the entries combined row by row and a check where it helps. If a product disagrees, compare the sizes first, then the individual sums of products.

Standards: N-VM.C.6, N-VM.C.7, N-VM.C.8, N-VM.C.9, N-VM.C.10 · **Level:** Grade 12 Precalculus

Problem 1 Data in a Matrix

The matrix has rows (48, 30, 25) and (36, 41, 52):

48	30	25
36	41	52

It has 2 rows and 3 columns, so it is 2×3 . The number in the second row and third column is 52, the bagels sold on Sunday.

Answer: **2×3 ; entry 52 (Sunday bagels)**

Problem 2 Adding and Subtracting

$A + B$:

-1	5
6	6

$A - B$:

7	-9
4	-6

Answer: **$A + B$ has rows $(-1, 5)$, $(6, 6)$; $A - B$ has rows $(7, -9)$, $(4, -6)$**

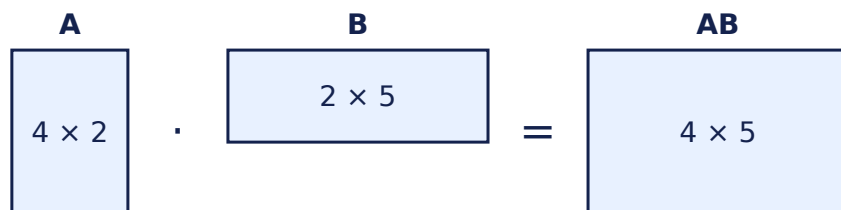
Problem 3 Scalar Multiple

Multiply every entry by -3 :

-6	3	-12
0	-15	6

Answer: **rows $(-6, 3, -12)$ and $(0, -15, 6)$**

Problem 4 Can the Product Be Found?



inner numbers must match

Sizes in the diagram: 4 by 2 and 2 by 5.

(a) The width of A (2) matches the height of B (2), so AB exists. It is as tall as A and as wide as B: 4×5 .

(b) For BA, the width of B (5) would have to match the height of A (4). It does not, so BA cannot be formed.

Answer: **AB is 4×5 ; BA does not exist**

Problem 5 A Determinant and Its Area

$\det = 8(6) - (-1)(3) = 48 + 3 = 51$. The area of the parallelogram is $|\det| = 51$ square units.

Answer: **$\det = 51$; area 51 square units**

Problem 6 Identity and Zero

The identity matrix leaves any matrix unchanged, and adding the zero matrix changes nothing, so both results equal A:

7	-3
2	9

Answer: **$IA = A + O = A$, with rows $(7, -3)$ and $(2, 9)$**

Problem 7 A 2×2 Product

Row 1: $2(5) + (-1)(-2) = 12$ and $2(0) + (-1)(1) = -1$. Row 2: $3(5) + 4(-2) = 7$ and $3(0) + 4(1) = 4$.

12	-1
7	4

Answer: **AB has rows $(12, -1)$ and $(7, 4)$**

Problem 8 Order Matters

AB :

8	9
6	15

BA :

4	5
2	19

The matrices differ, so matrix multiplication is not commutative here.

Answer: **AB has rows $(8, 9)$, $(6, 15)$ and BA has rows $(4, 5)$, $(2, 19)$; not equal**

Problem 9 A 2×3 Times a 3×2

A		B		AB																
<table><tr><td>1</td><td>0</td><td>2</td></tr><tr><td>-1</td><td>3</td><td>1</td></tr></table>	1	0	2	-1	3	1	$\times$	<table><tr><td>2</td><td>1</td></tr><tr><td>0</td><td>-3</td></tr><tr><td>4</td><td>5</td></tr></table>	2	1	0	-3	4	5	$=$	<table><tr><td>10</td><td>11</td></tr><tr><td>2</td><td>-5</td></tr></table>	10	11	2	-5
1	0	2																		
-1	3	1																		
2	1																			
0	-3																			
4	5																			
10	11																			
2	-5																			

$$\text{Entry (row 2, column 2)} = (-1)(1) + (3)(-3) + (1)(5) = -5$$

The shaded row and column combine into the shaded entry of the product.

AB is 2×2 . Row 1: $1(2) + 0(0) + 2(4) = 10$ and $1(1) + 0(-3) + 2(5) = 11$. Row 2: $(-1)(2) + 3(0) + 1(4) = 2$ and $(-1)(1) + 3(-3) + 1(5) = -5$.

Answer: **AB is 2×2 with rows $(10, 11)$ and $(2, -5)$**

Problem 10 Finding an Inverse

$\det A = 4(3) - 1(6) = 6$. Swap and negate: rows $(3, -1)$ and $(-6, 4)$. Divide by 6:

$\frac{1}{2}$	$-\frac{1}{6}$
-1	$\frac{2}{3}$

Check: row 1 times column 1 is $4(\frac{1}{2}) + 1(-1) = 1$; row 1 times column 2 is $4(-\frac{1}{6}) + 1(\frac{2}{3}) = 0$; the other entries work the same way, so $AA^{-1} = I$.

Answer: A^{-1} has rows $(\frac{1}{2}, -\frac{1}{6})$ and $(-1, \frac{2}{3})$

Problem 11 A 3×3 Product

Row 1: $1(2) + 0(1) + 2(-1) = 0$; $1(1) + 0(0) + 2(2) = 5$; $1(0) + 0(3) + 2(1) = 2$.

Row 2: $0 + 1 + 1 = 2$; $0 + 0 - 2 = -2$; $0 + 3 - 1 = 2$.

Row 3: $6 + 2 + 0 = 8$; $3 + 0 + 0 = 3$; $0 + 6 + 0 = 6$.

0	5	2
2	-2	2
8	3	6

Answer: rows **(0, 5, 2), (2, -2, 2), (8, 3, 6)**

Problem 12 No Inverse

A matrix has no inverse when its determinant is 0. $\det = 2(6) - k(3) = 12 - 3k$. Setting $12 - 3k = 0$ gives $k = 4$.

Check: with $k = 4$ the rows (2, 4) and (3, 6) are proportional (the second row is 1.5 times the first), and $\det = 12 - 12 = 0$.

Answer: **$k = 4$**

Problem 13 A Matrix Equation

Subtract A: $2X = B - A$, which has rows (8, -4) and (8, 4). Divide by 2:

4	-2
4	2

Check: $2(4) + 1 = 9$, $2(-2) + 3 = -1$, $2(4) - 5 = 3$, $2(2) + 7 = 11$.

Answer: **X has rows (4, -2) and (4, 2)**

Problem 14 Powers of a Matrix

$A^2 = A \cdot A$:

-1	0
0	-1

Then $A^4 = (A^2)^2$ has rows (1, 0) and (0, 1), the identity matrix I . Since $A^4 = I$, $A^{10} = (A^4)^2 \cdot A^2 = A^2 = -I$.

Answer: **A^2 has rows $(-1, 0)$, $(0, -1)$; $A^4 = I$; $A^{10} = A^2$**

Problem 15 Fundraiser Stands

Q has rows (40, 25, 15) and (30, 60, 10). P is the column (1.50, 0.75, 2.00).

Row 1: $40(1.50) + 25(0.75) + 15(2.00) = 60 + 18.75 + 30 = 108.75$. Row 2: $30(1.50) + 60(0.75) + 10(2.00) = 45 + 45 + 20 = 110$.

108.75
110

Each entry is the total revenue of one stand.

Answer: **Stand 1 earned \$108.75 and Stand 2 earned \$110.00**

Problem 16 Camp Enrollment

(a) Week 1 + Week 2:

20	32
36	24
40	60

(b) Multiply by 1.25:

25	40
45	30
50	75

Answer: **(a) rows (20, 32), (36, 24), (40, 60); (b) rows (25, 40), (45, 30), (50, 75)**

Problem 17 Secret Number Code

(a) K times the column (7, 4): $2(7) + 3(4) = 26$ and $1(7) + 4(4) = 23$. The code is (26, 23).

(b) $\det K = 2(4) - 3(1) = 5$, so K^{-1} has rows $(\frac{4}{5}, -\frac{3}{5})$ and $(-\frac{1}{5}, \frac{2}{5})$.

(c) $\frac{4}{5}(27) - \frac{3}{5}(26) = \frac{(108 - 78)}{5} = 6$ and $-\frac{1}{5}(27) + \frac{2}{5}(26) = \frac{(-27 + 52)}{5} = 5$.

Answer: **(26, 23); K^{-1} as above; the original pair is (6, 5)**

Problem 18 City and Suburb Moves

Population column after one year: P times (50,000; 30,000). City: $0.8(50,000) + 0.3(30,000) = 40,000 + 9,000 = 49,000$. Suburb: $0.2(50,000) + 0.7(30,000) = 10,000 + 21,000 = 31,000$.

After two years: City: $0.8(49,000) + 0.3(31,000) = 39,200 + 9,300 = 48,500$. Suburb: $0.2(49,000) + 0.7(31,000) = 9,800 + 21,700 = 31,500$.

The total stays 80,000.

Answer: **after one year 49,000 and 31,000; after two years 48,500 and 31,500**

Problem 19 Solving $AX = B$

$\det A = 3(5) - 2(4) = 7$, so $A^{-1} = \frac{1}{7}$ times rows (5, -2) and (-4, 3). Multiply both sides by A^{-1} on the left: $X = A^{-1}B$.

Row 1: $\frac{1}{7}(5 \cdot 9 - 2 \cdot 19, 5 \cdot (-6) - 2 \cdot (-8)) = \frac{1}{7}(7, -14) = (1, -2)$. Row 2: $\frac{1}{7}(-4 \cdot 9 + 3 \cdot 19, -4 \cdot (-6) + 3 \cdot (-8)) = \frac{1}{7}(21, 0) = (3, 0)$.

1	-2
3	0

Check: AX has first row $3(1) + 2(3) = 9$ and $3(-2) + 2(0) = -6$, matching B .

Answer: **X has rows (1, -2) and (3, 0)**

Problem 20 Inverse of a Product

AB has rows (5, 4) and (1, 2), with determinant $5(2) - 4(1) = 6$. So $(AB)^{-1} = \frac{1}{6}$ times rows (2, -4) and (-1, 5).

A^{-1} has rows (1, -2) and (0, 1). $\det B = 6$, so B^{-1} has rows ($\frac{1}{3}$, 0) and ($-\frac{1}{6}$, $\frac{1}{2}$).

$B^{-1}A^{-1}$: row 1 is ($\frac{1}{3}$, $-\frac{2}{3}$); row 2 is ($-\frac{1}{6}$, $\frac{1}{3} + \frac{1}{2}$) = ($-\frac{1}{6}$, $\frac{5}{6}$).

$\frac{1}{3}$	$-\frac{2}{3}$
$-\frac{1}{6}$	$\frac{5}{6}$

Both sides give the same matrix ($\frac{1}{6}$ times rows (2, -4) and (-1, 5)).

Answer: **both equal the matrix with rows ($\frac{1}{3}$, $-\frac{2}{3}$) and ($-\frac{1}{6}$, $\frac{5}{6}$)**