

Solving Systems and Transforming the Plane

Grade 12 – Precalculus math · Test Answer Key · Standards: A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12

Use the scoring notes for partial credit. Most items award a point for the correct matrix setup, a point for the determinant or inverse, and a point for the final answer.

Standards: A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12 · **Level:** Grade 12 Precalculus

Question 1 From Equations to Matrices (3 points)

(a) A has rows $(3, -5)$ and $(2, 1)$, X is the column (x, y) and B is the column $(-4, 7)$.

(b) $\det A = 3(1) - (-5)(2) = 13$.

(c) Since $13 \neq 0$, A is invertible and the system has exactly one solution.

Answer: **(a) as stated; (b) 13; (c) yes, unique**

Scoring: 1 point for each part.

Question 2 Solve Using an Inverse (3 points)

A has rows $(3, 2)$ and $(7, 4)$; $\det A = 12 - 14 = -2$. So $A^{-1} = -\frac{1}{2}$ times rows $(4, -2)$ and $(-7, 3)$.

$X = -\frac{1}{2}(4 \cdot 4 - 2 \cdot 6, -7 \cdot 4 + 3 \cdot 6) = -\frac{1}{2}(4, -10) = (-2, 5)$.

Check: $3(-2) + 2(5) = 4$ and $7(-2) + 4(5) = 6$.

Answer: **$x = -2$ and $y = 5$**

Scoring: 1 point for A^{-1} , 1 point for the multiplication, 1 point for the check or the correct solution.

Question 3 Type of Solution Set (2 points)

(a) $\det = 4(9) - (-6)(-6) = 36 - 36 = 0$.

(b) The second equation is -1.5 times the first ($4(-1.5) = -6$, $-6(-1.5) = 9$, $10(-1.5) = -15$), so both equations describe the same line. There are infinitely many solutions.

Answer: **$\det = 0$; infinitely many solutions**

Scoring: 1 point for each part.

Question 4 Images of Points (2 points)

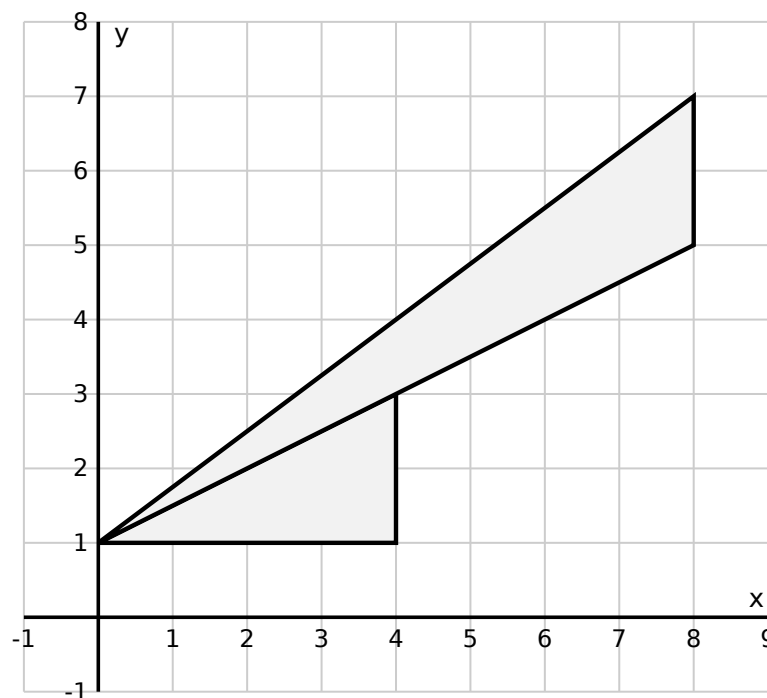
$(4, -2)$: $2(4) + (-1)(-2) = 10$ and $0(4) + 3(-2) = -6$, giving $(10, -6)$.

$(-1, 5)$: $2(-1) + (-1)(5) = -7$ and $0 + 3(5) = 15$, giving $(-7, 15)$.

Answer: **$(10, -6)$ and $(-7, 15)$**

Scoring: 1 point for each image.

Question 5 A Triangle Transformed (3 points)



The original triangle and its image.

(a) $(0, 1)$ goes to $(2(0) + 0, 1(0) + 1(1)) = (0, 1)$. $(4, 1)$ goes to $(8, 4 + 1) = (8, 5)$. $(4, 3)$ goes to $(8, 4 + 3) = (8, 7)$.

(b) $\det = 2(1) - 0(1) = 2$. The original area is $\frac{1}{2}(4)(2) = 4$, so the image area is $4 \cdot 2 = 8$ square units.

Answer: **$(0, 1)$, $(8, 5)$, $(8, 7)$; $\det = 2$; area 8**

Scoring: 1 point for part (a); 1 point for the determinant and 1 point for the area.

Question 6 Matrices from Descriptions (2 points)

(a) $(1(-3) + 0, 0 - 1(7)) = (-3, -7)$.

(b) A 90° clockwise rotation sends $(1, 0)$ to $(0, -1)$ and $(0, 1)$ to $(1, 0)$. Writing these as columns gives rows $(0, 1)$ and $(-1, 0)$.

Answer: **(a) $(-3, -7)$; (b) rows $(0, 1)$ and $(-1, 0)$**

Scoring: 1 point for each part.

Question 7 Snack Bar Prices (3 points)

(a) A has rows $(6, 2)$ and $(5, 3)$, $X = (b, j)$ and $B = (18, 17)$.

(b) $\det A = 18 - 10 = 8$, so $A^{-1} = \frac{1}{8}$ times rows $(3, -2)$ and $(-5, 6)$. $b = \frac{(3 \cdot 18 - 2 \cdot 17)}{8} = \frac{20}{8} = 2.50$ and $j = \frac{(-5 \cdot 18 + 6 \cdot 17)}{8} = \frac{12}{8} = 1.50$.

Check: $6(2.50) + 2(1.50) = 18$ and $5(2.50) + 3(1.50) = 17$.

Answer: **bagels \$2.50 and juice \$1.50**

Scoring: 1 point for part (a); 2 points for part (b): 1 for the inverse and 1 for the solution.

Question 8 Singular System (2 points)

$\det = 3(4) - k(2) = 12 - 2k = 0$ gives $k = 6$.

With $k = 6$: $3x + 6y = 9$ simplifies to $x + 2y = 3$, and $2x + 4y = 5$ simplifies to $x + 2y = 2.5$. The same left side cannot equal 3 and 2.5, so the lines are parallel and there is no solution.

Answer: **$k = 6$; no solution**

Scoring: 1 point for k , 1 point for the classification.