

Solving Systems and Transforming the Plane

Grade 12 – Precalculus math · Worksheet · Standards: A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12

Name: _____ Date: _____ Class: _____

These problems cover writing systems as matrix equations, solving with inverse matrices, and using 2×2 matrices to transform points and areas. Standards: A-REI.C.8, A-REI.C.9, N-VM.C.11 and N-VM.C.12.

The last problems bring in a food truck, a chemistry mixture, a 3×3 ticket system and a robot work zone.

Standards: A-REI.C.8, A-REI.C.9, N-VM.C.11, N-VM.C.12 · **Level:** Grade 12 Precalculus

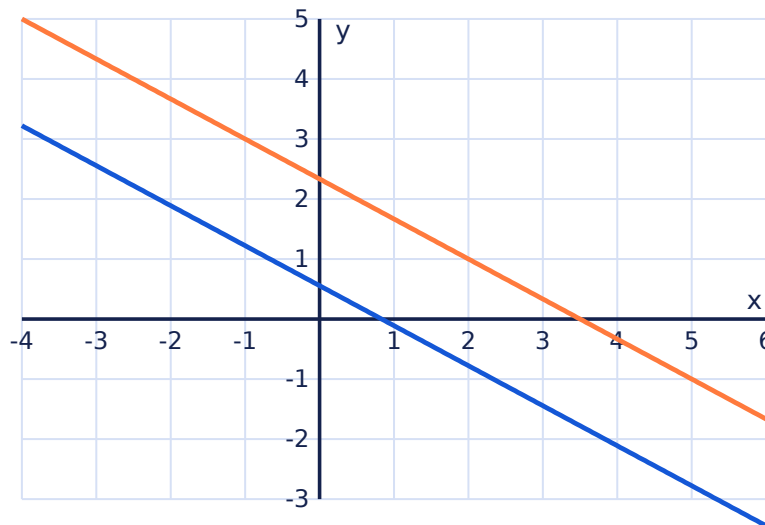
Problem 1 Writing a Matrix Equation (easy)

Write the system $4x - y = 9$ and $2x + 5y = -3$ in the form $AX = B$. Name A , X and B .

Problem 2 Reading a Matrix Equation (easy)

Write the matrix equation as two equations, then check whether $(1, -2)$ is a solution. A has rows $(2, -3)$ and $(5, 1)$, X is the column (x, y) , and B is the column $(8, 3)$.

Problem 3 Is There One Solution? (easy)



Two lines on the same grid.

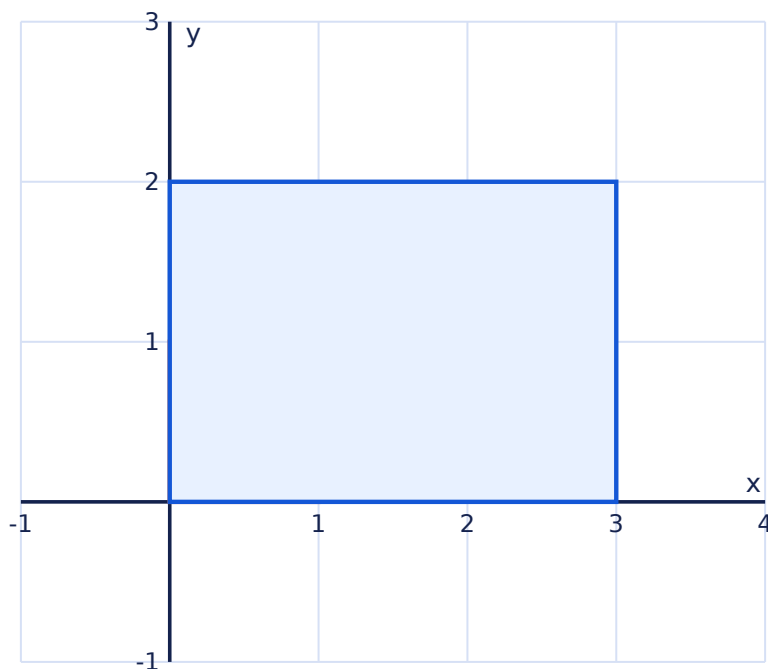
Decide whether the system $6x + 9y = 5$ and $2x + 3y = 7$ has a unique solution. Use the determinant.

Problem 4 Matrix Times a Point (easy)

Find the product of the matrix with rows $(3, -1)$ and $(2, 4)$ and the column $(5, -2)$.

Problem 5 Reflecting a Point (easy)

The matrix with rows $(-1, 0)$ and $(0, 1)$ reflects points in the y -axis. Find the image of $(6, -4)$.

Problem 6 Stretching the Unit Square (easy)

The unit square and its image under the stretch matrix.

The matrix with rows $(3, 0)$ and $(0, 2)$ transforms the unit square into the rectangle shown. Find its determinant and the area of the image.

Problem 7 Solving with an Inverse (medium)

Solve $4x + 3y = 6$ and $5x + 2y = 11$ using an inverse matrix.

Problem 8 A Negative Solution (medium)

Solve $3x + 4y = -7$ and $2x - y = -12$ with matrices.

Problem 9 A Fractional Solution (medium)

Solve $2x + 3y = 5$ and $4x - y = 6$ using $X = A^{-1}B$.

Problem 10 Two Moves in a Row (medium)

Apply the rotation with rows $(0, -1)$ and $(1, 0)$ to the point $(-3, 5)$, then reflect the result in the x -axis using the matrix with rows $(1, 0)$ and $(0, -1)$. Then find one matrix that does both moves

at once.

Problem 11 Area After a Transformation (medium)

A triangle with area 6 square units is transformed by the matrix with rows $(4, -2)$ and $(1, 3)$. Find the area of the image.

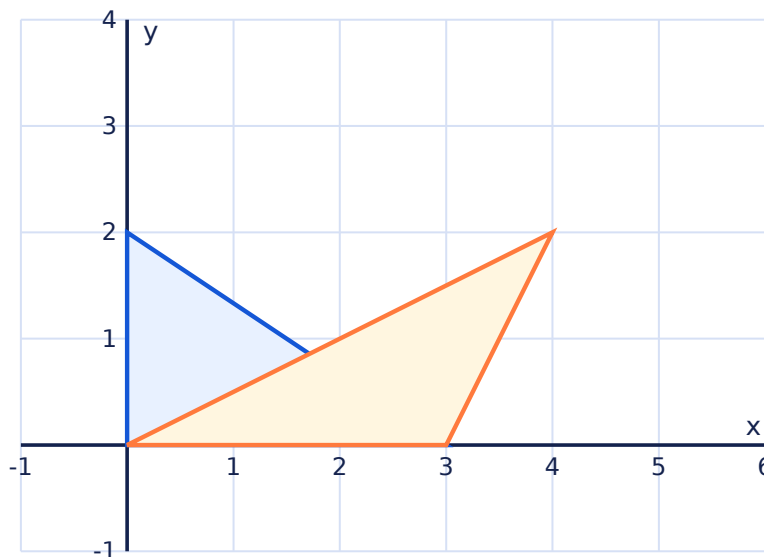
Problem 12 Building a Scaling Matrix (medium)

Write the matrix that doubles every x -coordinate and triples every y -coordinate. Find the image of $(2, -1)$ and the area of the image of the unit square.

Problem 13 A Matrix That Flattens (medium)

The matrix with rows $(2, 4)$ and $(3, 6)$ is applied to the plane. Find its determinant, the images of $(1, 0)$ and $(0, 1)$, and describe what happens to the plane.

Problem 14 A Shear (medium)



The triangle and its image after the shear.

The matrix with rows $(1, 2)$ and $(0, 1)$ shears the triangle with vertices $(0, 0)$, $(3, 0)$ and $(0, 2)$. Find the image of each vertex and compare the two areas.

Problem 15 Food Truck Prices (hard)

A food truck sold 18 wraps and 12 bowls on Friday for \$246, and 25 wraps and 20 bowls on Saturday for \$365. Set up a matrix equation and use an inverse to find the price of a wrap and of a bowl.

Problem 16 Mixing Solutions (hard)

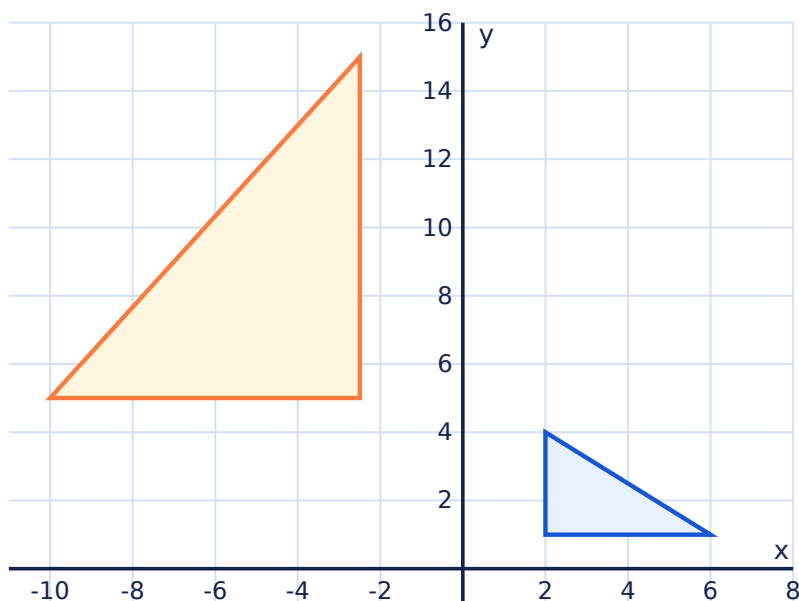
A chemistry club mixes a 20% acid solution with a 50% acid solution to make 12 liters of 38% acid solution. Let x and y be the liters of each. Write a matrix equation and solve it with an

inverse.

Problem 17 Theater Ticket Prices (hard)

A theater sells adult, child and senior tickets at prices a , c and s . Three shows gave these totals: Show 1 sold 40 adult, 30 child and 10 senior tickets for \$810. Show 2 sold 25, 45 and 20 for \$840. Show 3 sold 30, 20 and 30 for \$790. Write the system as $AX = B$ (divide each equation by 5 first) and use technology to find the three prices.

Problem 18 Robot Arm Zone (hard)



The work zone before and after the transformation.

A robot's work zone is the triangle with vertices $(2, 1)$, $(6, 1)$ and $(2, 4)$. A controller applies the matrix with rows $(0, -2.5)$ and $(2.5, 0)$, which rotates by 90° counterclockwise and scales by 2.5. Find the image of each vertex and the area of the new zone two ways.

Problem 19 Transformation from Images (hard)

A transformation sends $(1, 0)$ to $(3, 1)$ and $(0, 1)$ to $(-1, 2)$. (a) Write its matrix. (b) Find the image of $(2, 5)$. (c) Find the determinant and say how areas change. (d) Find the point that is sent to $(8, 5)$.

Problem 20 When Is There No Unique Solution? (hard)

Consider $2x + ky = 8$ and $3x + 6y = 12$. (a) Find the value of k for which the system has no unique solution. (b) For that k , does the system have no solution or infinitely many? (c) For $k = 5$, solve with an inverse.