

Polar Coordinates and Parametric Curves

Grade 12 – Precalculus math · Test Answer Key · Standards: N-CN.B.4, F-IF.C.7

Use the scoring notes for partial credit: usually one point per correct coordinate, equation or measurement. Full marks need the exact form requested and a correct angle or quadrant.

Standards: N-CN.B.4, F-IF.C.7 · **Level:** Grade 12 Precalculus

Question 1 Polar to Rectangular (2 points)

$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ and $\sin \frac{7\pi}{6} = -\frac{1}{2}$. So $x = -5\sqrt{3}$ and $y = -5$.

Answer: **$(-5\sqrt{3}, -5)$**

Scoring: 1 point for each coordinate.

Question 2 Rectangular to Polar (2 points)

$r = \sqrt{4 + 12} = 4$. The reference angle is $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ and the point is in Quadrant II, so $\theta = \frac{2\pi}{3}$.

Answer: **$(4, \frac{2\pi}{3})$**

Scoring: 1 point for r , 1 point for θ .

Question 3 Converting Equations (2 points)

(a) Multiply by r : $r^2 = -4r \sin \theta$, so $x^2 + y^2 = -4y$ and $x^2 + (y + 2)^2 = 4$.

(b) $r = \frac{6}{(\cos \theta)}$ means $r \cos \theta = 6$, so $x = 6$.

Answer: **(a) $x^2 + (y + 2)^2 = 4$; (b) $x = 6$**

Scoring: 1 point for each part.

Question 4 Reading a Rose (2 points)

(a) The figure has 4 petals, each reaching the 6th ring, so each is 6 units long. (This is $r = 6 \cos 2\theta$.)

(b) An odd n gives n petals, so $n = 7$ and $a = 2$.

Answer: **(a) 4 petals, 6 units each; (b) $r = 2 \cos 7\theta$ (or $2 \sin 7\theta$)**

Scoring: 1 point for part (a), 1 point for part (b).

Question 5 Cardioid Values (2 points)

$$r(0) = 2, r(\pi/2) = 4, r(\pi) = 2, r(3\pi/2) = 0.$$

The graph touches the pole where $r = 0$, at $\theta = 3\pi/2$.

Answer: **2, 4, 2, 0; the pole at $\theta = 3\pi/2$**

Scoring: 1 point for the four values, 1 point for the angle.

Question 6 Eliminating the Parameter (2 points)

$$(a) t = (x+2)/3, \text{ so } y = 1 - (2(x+2))/3 = -(2x+1)/3.$$

(b) $\cos t = (x-1)/3$ and $\sin t = (y+2)/3$, so $(x-1)^2 + (y+2)^2 = 9$, a circle with center $(1, -2)$ and radius 3.

Answer: **(a) $y = -(2x+1)/3$; (b) $(x-1)^2 + (y+2)^2 = 9$**

Scoring: 1 point for each part.

Question 7 A Thrown Water Balloon (3 points)

(a) $x = 15$ and $y = 4 + 28 - 16 = 16$. The position is $(15, 16)$.

(b) The vertex is at $t = 28/32 = 0.875$, where $y = 4 + 24.5 - 12.25 = 16.25$ ft.

(c) Solve $4t^2 - 7t - 1 = 0$: $t = (7 + \sqrt{65})/8 \approx 1.883$ s, so $x \approx 15 \cdot 1.883 \approx 28.2$ ft.

Answer: **(15, 16); 16.25 ft; about 28.2 ft**

Scoring: 1 point for each part.

Question 8 Distance on a Radar Screen (3 points)

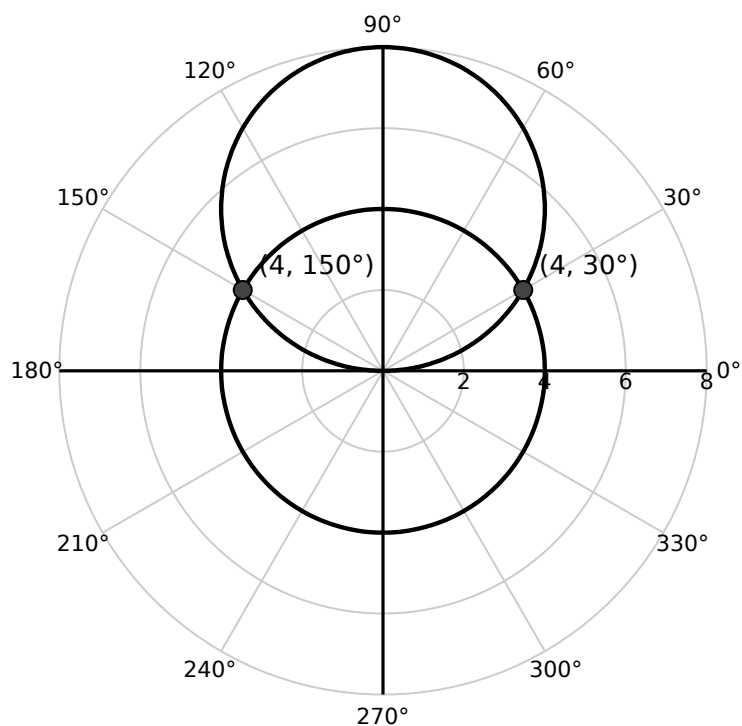
(a) $80^\circ - 20^\circ = 60^\circ$.

(b) Law of cosines: $d^2 = 8^2 + 5^2 - 2(8)(5) \cos 60^\circ = 64 + 25 - 40 = 49$, so $d = 7$.

Answer: **60°; 7 miles**

Scoring: 1 point for part (a); 1 point for the law of cosines setup and 1 point for the distance.

Question 9 Intersecting Circles (2 points)



The circles meet at $(4, \pi/6)$ and $(4, 5\pi/6)$.

Set $4 = 8 \sin \theta$, so $\sin \theta = \frac{1}{2}$ and $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

Answer: **$(4, \frac{\pi}{6})$ and $(4, \frac{5\pi}{6})$**

Scoring: 1 point for each point.