

Polar Coordinates and Parametric Curves

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: N-CN.B.4, F-IF.C.7

Every answer shows the conversion formulas used and a check where it helps. Compare your quadrant choices for angles and your algebra when completing the square or eliminating a parameter.

Standards: N-CN.B.4, F-IF.C.7 · **Level:** Grade 12 Precalculus

Problem 1 Polar to Rectangular

$$x = 6 \cos 60^\circ = 6 \cdot \frac{1}{2} = 3 \text{ and } y = 6 \sin 60^\circ = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}.$$

Answer: **(3, 3√3)**

Problem 2 A Negative Radius

$$x = -3 \cos \frac{\pi}{4} = -3 \cdot \frac{\sqrt{2}}{2} = -\frac{3\sqrt{2}}{2} \text{ and } y = -3 \sin \frac{\pi}{4} = -\frac{3\sqrt{2}}{2}.$$

This is the same point as $(3, \frac{5\pi}{4})$, in Quadrant III.

Answer: **$(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}) \approx (-2.12, -2.12)$**

Problem 3 Rectangular to Polar

$$r = \sqrt{27 + 9} = 6. \text{ The reference angle is } \tan^{-1}(\frac{3}{\sqrt{3}}) = \frac{\pi}{6}, \text{ and the point is in Quadrant II, so } \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Answer: **$(6, \frac{5\pi}{6})$**

Problem 4 A Point in Quadrant III

$$r = \sqrt{36 + 36} = 6\sqrt{2}. \text{ The reference angle is } \frac{\pi}{4} \text{ and the point is in Quadrant III, so } \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}.$$

Answer: **$(6\sqrt{2}, \frac{5\pi}{4})$**

Problem 5 Reading a Polar Grid

(a) A is on the 4th ring along the 210° ray: **(4, 210°)**.

(b) Add a full turn: $(4, 570^\circ)$. Use a negative radius and add 180° (or subtract it): $(-4, 30^\circ)$.

(c) $x = 4 \cos 210^\circ = -2\sqrt{3}$ and $y = 4 \sin 210^\circ = -2$.

Answer: **$(4, 210^\circ)$, also $(4, 570^\circ)$ and $(-4, 30^\circ)$; rectangular $(-2\sqrt{3}, -2)$**

Problem 6 A Circle from a Sine

Multiply by r : $r^2 = 8r \sin \theta$, so $x^2 + y^2 = 8y$. Move and complete the square: $x^2 + (y - 4)^2 = 16$.

The graph is a circle with center $(0, 4)$ and radius 4.

Answer: **$x^2 + (y - 4)^2 = 16$**

Problem 7 A Line in Polar Form

Substitute $x = r \cos \theta$ and $y = r \sin \theta$: $2r \cos \theta - 5r \sin \theta = 10$. Factor out r : $r(2 \cos \theta - 5 \sin \theta) = 10$.

Answer: **$r = \frac{10}{2 \cos \theta - 5 \sin \theta}$**

Problem 8 A Circle Through the Pole

Replace $x^2 + y^2$ with r^2 and x with $r \cos \theta$: $r^2 + 6r \cos \theta = 0$. Divide by r : $r + 6 \cos \theta = 0$.

Answer: **$r = -6 \cos \theta$**

Problem 9 Analyzing a Cardioid

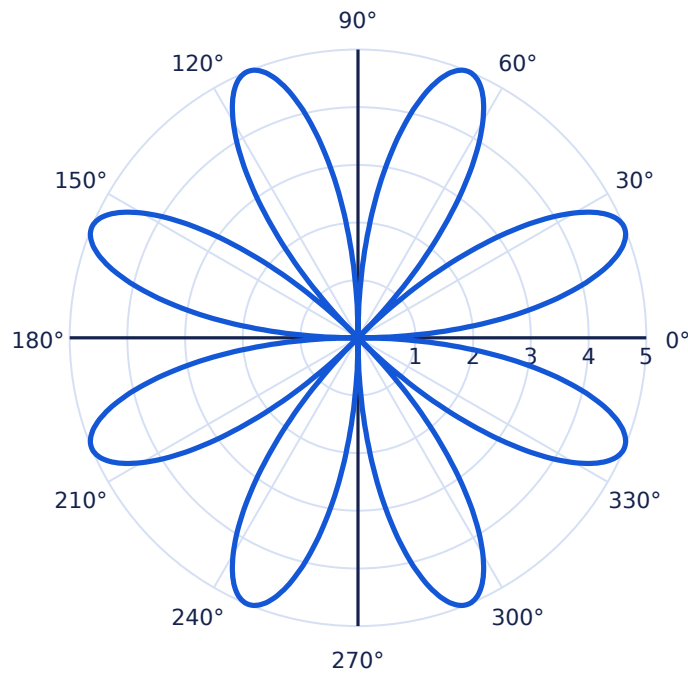
(a) Since $a = b = 3$, it is a cardioid.

(b) $r = 6$ at 0 , $r = 3$ at $\frac{\pi}{2}$, $r = 0$ at π , $r = 3$ at $\frac{3\pi}{2}$.

(c) The maximum distance is 6, at $\theta = 0$, and the graph reaches the pole at $\theta = \pi$.

Answer: **cardioid; values 6, 3, 0, 3; farthest 6 units, cusp at the pole at $\theta = \pi$**

Problem 10 An Eight-Petal Rose



The rose curve.

Since 4 is even, the rose has $2 \cdot 4 = 8$ petals. Each petal has length $|5| = 5$.

The first tip occurs where $\sin 4\theta = 1$, so $4\theta = \frac{\pi}{2}$ and $\theta = \frac{\pi}{8}$.

Answer: **8 petals, each 5 units long, first tip at $\theta = \frac{\pi}{8}$**

Problem 11 A Limaçon with a Loop

Since $a = 2 < b = 4$, the limaçon has an inner loop. $r = 0$ when $\cos \theta = -\frac{1}{2}$, so $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

The largest r occurs at $\theta = 0$: $r = 2 + 4 = 6$.

Answer: **inner loop; pole at $\theta = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$; maximum $r = 6$**

Problem 12 A Three-Petal Rose

Since 3 is odd, the rose has 3 petals. Tips occur where $|\cos 3\theta| = 1$, so $3\theta = 0, \pi, 2\pi$ and $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$.

At $\theta = 0$, $r = 4$. At $\theta = \frac{\pi}{3}$, $r = -4$, which is the point $(4, \frac{4\pi}{3})$. At $\theta = \frac{2\pi}{3}$, $r = 4$.

Answer: **3 petals; tips at distance 4 in the directions $0, \frac{2\pi}{3}$ and $\frac{4\pi}{3}$**

Problem 13 Eliminating a Parameter

Solve $x = 2t + 1$ for t : $t = \frac{(x-1)}{2}$. Then $y = \frac{(x-1)}{2} - 3 = \frac{x}{2} - \frac{7}{2}$.

At $t = 0$ the point is $(1, -3)$. At $t = 5$ it is $(11, 2)$. The path is the line segment between them.

Answer: $y = \frac{x}{2} - \frac{7}{2}$, from **(1, -3) to (11, 2)**

Problem 14 An Elliptical Path

$\cos t = \frac{(x-2)}{5}$ and $\sin t = \frac{(y+1)}{3}$. Using $\cos^2 t + \sin^2 t = 1$: $\frac{((x-2)^2)}{25} + \frac{((y+1)^2)}{9} = 1$.

The path is an ellipse with center $(2, -1)$, horizontal semi-axis 5 and vertical semi-axis 3.

Answer: $\frac{((x-2)^2)}{25} + \frac{((y+1)^2)}{9} = 1$, center **(2, -1)**

Problem 15 Is It a Parabola?

Multiply: $r + r \cos \theta = 4$, so $r = 4 - x$. Square both sides: $x^2 + y^2 = 16 - 8x + x^2$.

The x^2 terms cancel: $y^2 = 16 - 8x$. This is a parabola opening to the left with vertex $(2, 0)$.

Answer: $y^2 = 16 - 8x$, a parabola

Problem 16 The Kicked Ball

(a) $x = 9$ and $y = 5 + 12 - 4 = 13$. The position is $(9, 13)$.

(b) The vertex of y is at $t = \frac{24}{32} = 0.75$: $y = 5 + 18 - 9 = 14$ ft.

(c) Solve $-16t^2 + 24t + 5 = 0$: $t = \frac{(24 + \sqrt{896})}{32} \approx 1.685$ s. Then $x = 18(1.685) \approx 30.3$ ft.

Answer: **(9, 13); 14 ft; about 30.3 ft**

Problem 17 Two Boats on Radar

(a) $150^\circ - 30^\circ = 120^\circ$.

(b) Law of cosines: $d^2 = 9^2 + 7^2 - 2(9)(7) \cos 120^\circ = 81 + 49 + 63 = 193$, so $d = \sqrt{193} \approx 13.9$ mi.

Check with coordinates: $A \approx (7.79, 4.5)$ and $B \approx (-6.06, 3.5)$. The differences are 13.85 and 1.0, and $\sqrt{(13.85)^2 + 1^2} \approx 13.9$.

Answer: **120°; about 13.9 miles**

Problem 18 Microphone Pickup Pattern

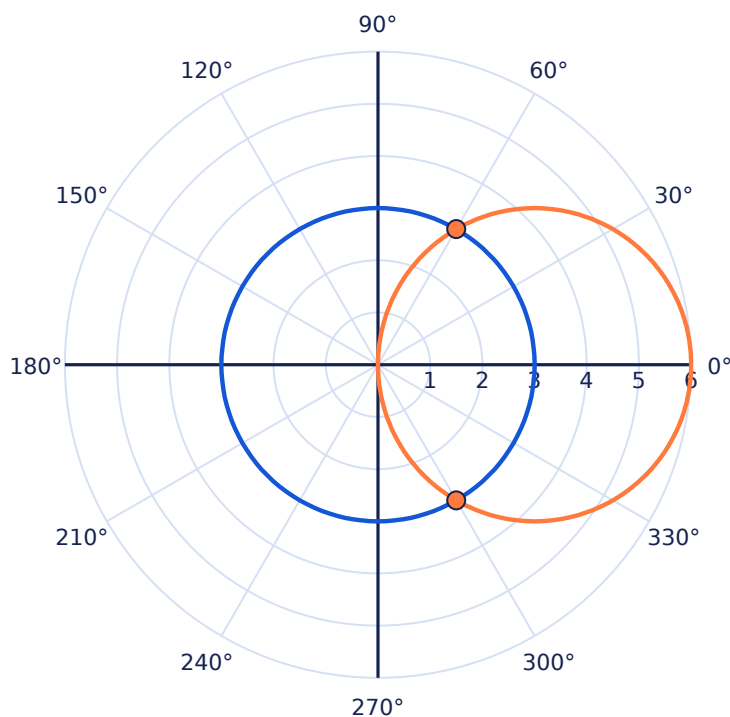
(a) $a = b = 4$, so the pattern is a cardioid.

(b) At $\theta = \frac{\pi}{2}$: $r = 4 + 4 = 8$ ft. At $\theta = \frac{3\pi}{2}$: $r = 4 - 4 = 0$, so the microphone picks up nothing from directly behind.

(c) At $\theta = \frac{\pi}{6}$: $r = 4 + 4 \cdot \frac{1}{2} = 6$ ft.

Answer: **cardioid; 8 ft ahead and 0 ft behind; 6 ft at $\theta = \frac{\pi}{6}$**

Problem 19 Where Two Curves Meet



The circles meet at angles $\pm 60^\circ$.

Set $3 = 6 \cos \theta$, so $\cos \theta = \frac{1}{2}$ and $\theta = \pm \frac{\pi}{3}$. The points are $(3, \frac{\pi}{3})$ and $(3, -\frac{\pi}{3})$.

Rectangular: $x = 3 \cos \frac{\pi}{3} = \frac{3}{2}$ and $y = \pm 3 \sin \frac{\pi}{3} = \pm \frac{3\sqrt{3}}{2}$.

Check: $x^2 + y^2 = \frac{9}{4} + \frac{27}{4} = 9$ and $(x - 3)^2 + y^2 = \frac{9}{4} + \frac{27}{4} = 9$.

Answer: **$(3, \pm \frac{\pi}{3})$, or $(\frac{3}{2}, \pm \frac{3\sqrt{3}}{2})$**

Problem 20 Ferris Wheel Motion

(a) $\cos(\frac{t}{5}) = \frac{x}{20}$ and $\sin(\frac{t}{5}) = \frac{(y - 24)}{20}$. So $x^2 + (y - 24)^2 = 400$, a circle with center $(0, 24)$ and radius 20.

(b) At $t = 5$ the angle is 1 radian: $x = 20 \cos 1 \approx 10.8$ and $y = 24 + 20 \sin 1 \approx 40.8$.

(c) A full turn needs $\frac{t}{5} = 2\pi$, so $t = 10\pi \approx 31.4$ s.

Answer: $x^2 + (y - 24)^2 = 400$; **(10.8, 40.8)**; **$10\pi \approx 31.4$ s**