

Polar Coordinates and Parametric Curves

Grade 12 – Precalculus math · Worksheet · Standards: N-CN.B.4, F-IF.C.7

Name: _____ Date: _____ Class: _____

These problems cover plotting and converting polar points, switching equations between polar and rectangular form, graphing circles, roses and limaçons, and parametric motion. Standards: N-CN.B.4 and F-IF.C.7.

Work from direct conversions to projectile, radar and Ferris wheel models. Give exact values with π when possible.

Standards: N-CN.B.4, F-IF.C.7 · **Level:** Grade 12 Precalculus

Problem 1 Polar to Rectangular (easy)

Convert the polar point $(6, 60^\circ)$ to rectangular coordinates.

Problem 2 A Negative Radius (easy)

Convert the polar point $(-3, \frac{\pi}{4})$ to rectangular coordinates.

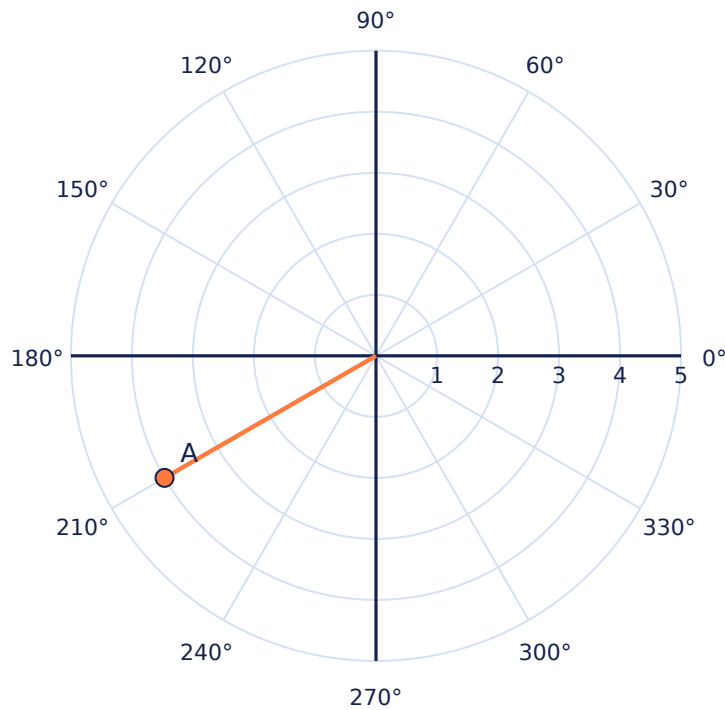
Problem 3 Rectangular to Polar (easy)

Write $(-3\sqrt{3}, 3)$ in polar form with $r > 0$ and $0 \leq \theta < 2\pi$.

Problem 4 A Point in Quadrant III (easy)

Write $(-6, -6)$ in polar form with $r > 0$ and $0 \leq \theta < 2\pi$.

Problem 5 Reading a Polar Grid (easy)



The point A on a polar grid.

Point A is shown on the polar grid. (a) Write A as (r, θ) with $r > 0$. (b) Give two other polar names for A. (c) Find its rectangular coordinates.

Problem 6 A Circle from a Sine (easy)

Convert $r = 8 \sin \theta$ to rectangular form and describe the graph.

Problem 7 A Line in Polar Form (medium)

Write the line $2x - 5y = 10$ in polar form, solving for r .

Problem 8 A Circle Through the Pole (medium)

Convert $x^2 + y^2 + 6x = 0$ to polar form, solving for r (ignore the pole).

Problem 9 Analyzing a Cardioid (medium)

Consider $r = 3 + 3 \cos \theta$. (a) Name the type of graph. (b) Find r at $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$. (c) State the farthest distance from the pole and where the graph touches the pole.

Problem 10 An Eight-Petal Rose (medium)

Describe the graph of $r = 5 \sin 4\theta$: the number of petals, the length of each petal, and the first angle at which a petal tip occurs for $\theta \geq 0$.

Problem 11 A Limaçon with a Loop (medium)

Show that $r = 2 + 4 \cos \theta$ has an inner loop. Find the angles where the graph passes through

the pole and the largest value of r .

Problem 12 A Three-Petal Rose (medium)

Find the tip of each petal of $r = 4 \cos 3\theta$ for $0 \leq \theta < \pi$, and state how many petals the graph has.

Problem 13 Eliminating a Parameter (medium)

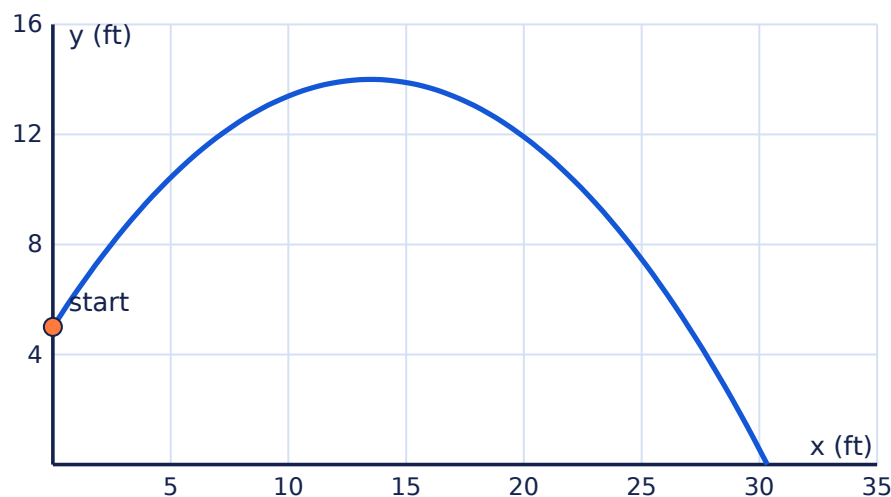
A particle moves with $x = 2t + 1$ and $y = t - 3$ for $0 \leq t \leq 5$. Eliminate the parameter, then give the starting and ending points.

Problem 14 An Elliptical Path (medium)

A drone circles a pond along $x = 2 + 5 \cos t$, $y = -1 + 3 \sin t$, for $0 \leq t < 2\pi$ (distances in yards). Find the rectangular equation of the path, its center, and its semi-axes.

Problem 15 Is It a Parabola? (hard)

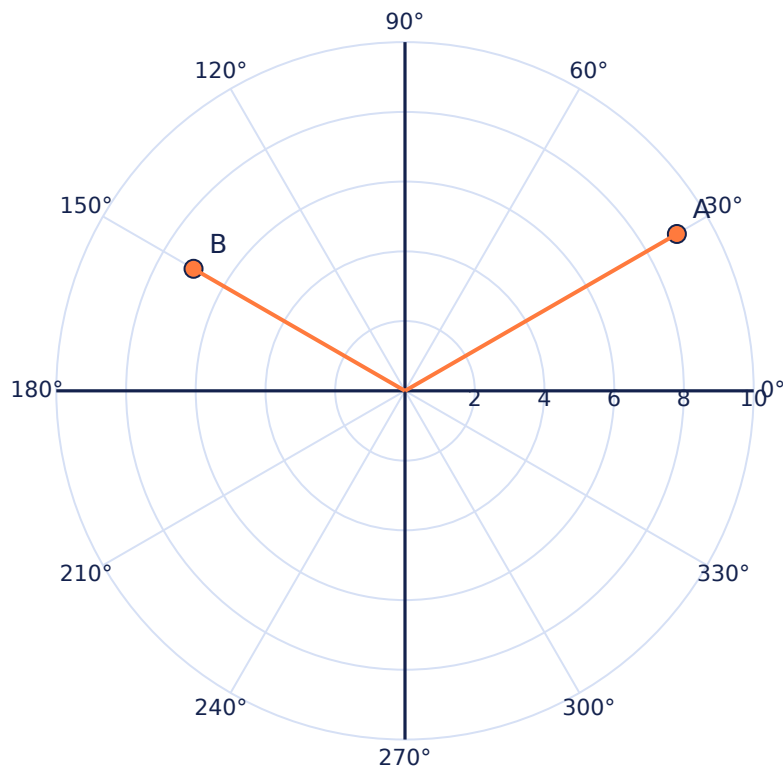
Convert $r = \frac{4}{(1 + \cos \theta)}$ to rectangular form and identify the graph.

Problem 16 The Kicked Ball (hard)

Path of the kicked ball. Not all labeled values are shown.

A ball is kicked with $x = 18t$ and $y = 5 + 24t - 16t^2$, in feet, with t in seconds. (a) Find the ball's position at $t = 0.5$. (b) Find its maximum height. (c) Find how far it travels horizontally before hitting the ground, to the nearest tenth of a foot.

Problem 17 Two Boats on Radar (hard)



Radar screen: rings every 2 miles.

A radar screen shows boat A at polar coordinates (9 mi, 30°) and boat B at (7 mi, 150°). (a) Find the angle at the radar between the two boats. (b) Find the distance between them to the nearest tenth of a mile.

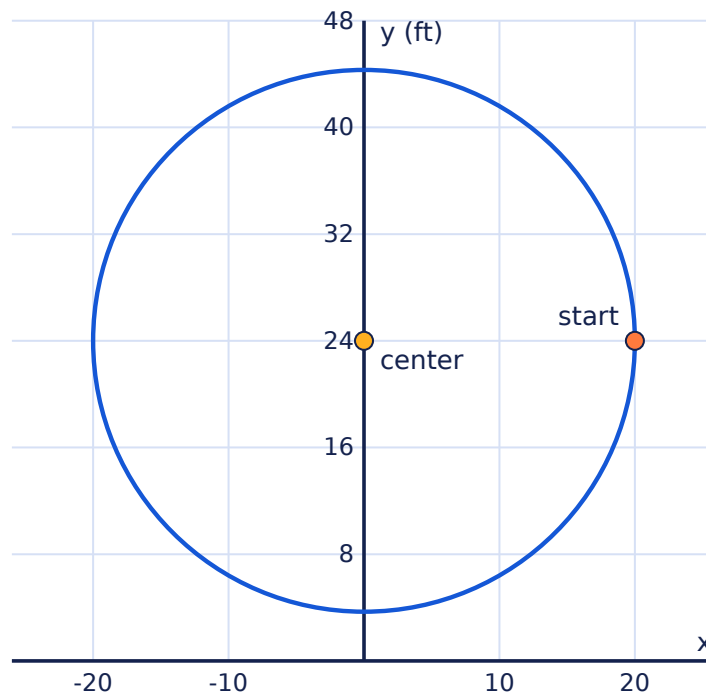
Problem 18 Microphone Pickup Pattern (hard)

A studio microphone picks up sound in the pattern $r = 4 + 4 \sin \theta$ (in feet of effective range), where $\theta = \frac{\pi}{2}$ points straight at the performer. (a) Name the shape. (b) Find the range straight ahead and straight behind ($\theta = \frac{3\pi}{2}$). (c) Find the range at $\theta = \frac{\pi}{6}$.

Problem 19 Where Two Curves Meet (hard)

Find the points where $r = 3$ and $r = 6 \cos \theta$ intersect. Give them in polar form for $-\pi < \theta \leq \pi$, then in rectangular form.

Problem 20 Ferris Wheel Motion (hard)



The wheel: radius 20 ft, center 24 ft above the ground.

A rider on a Ferris wheel moves with $x = 20 \cos(\frac{t}{5})$ and $y = 24 + 20 \sin(\frac{t}{5})$, in feet, t in seconds.
 (a) Eliminate the parameter. (b) Find the rider's position at $t = 5$ to the nearest tenth. (c) Find the time of one full turn.