

Polynomial and Rational Functions: Zeros and Asymptotes

Grade 12 – Precalculus math · Test Answer Key · Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9

Each solution shows the work with a scoring note. Award method points separately from final-answer points, so a small arithmetic slip does not erase credit for a correct sign chart or division.

Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · **Level:** Grade 12 Precalculus

Question 1 Find All Zeros (2 points)

Candidates $\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2$. Since $P(2) = 16 + 4 - 26 + 6 = 0$, divide to get $2x^2 + 5x - 3 = (2x - 1)(x + 3)$.

Zeros: **2, 1/2, -3.**

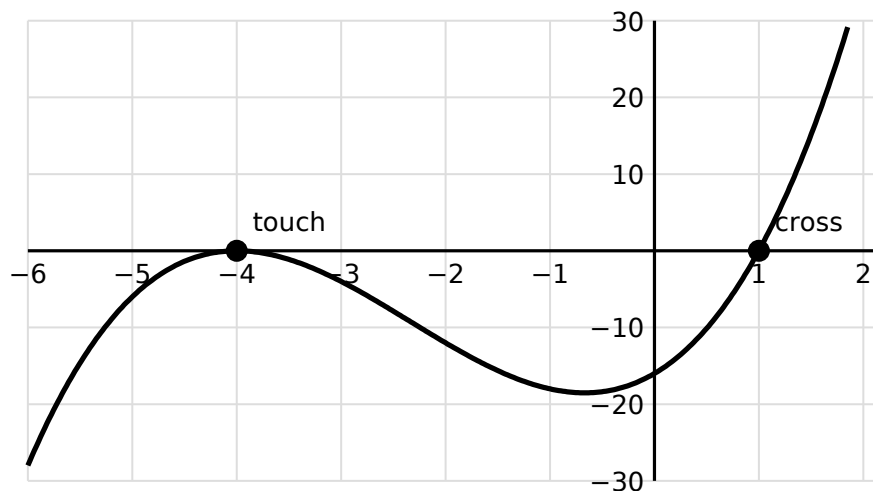
Scoring: 1 point for a valid first zero with work; 1 point for the remaining two zeros.

Question 2 Conjugate Zeros and Intercepts (2 points)

- a. **$1 - 3i$ and $2i$.**
- b. All four zeros are non-real, so the graph has **0 x-intercepts**.

Scoring: 1 point per part.

Question 3 Zeros and End Behavior from a Graph (2 points)



The touch point has even multiplicity.

- -4 has multiplicity 2 (touch) and 1 has multiplicity 1 (cross).
- Odd degree, positive leading coefficient: falls on the left, rises on the right. $P(x) = (x + 4)^2(x - 1)$ (check: $P(0) = -16$).

Scoring: 1 point per part.

Question 4 Slant and Vertical Asymptotes (2 points)

Divide: $4x^2 - 5x + 3 = (4x - 1)(x - 1) + 2$. The remainder is 2, so $f(x) = 4x - 1 + \frac{2}{(x - 1)}$.

Slant asymptote $y = 4x - 1$; vertical asymptote $x = 1$.

Scoring: 1 point for each asymptote.

Question 5 Polynomial Inequality (2 points)

Factor: $x(2x + 5)(x - 3) \leq 0$. Critical numbers $-5/2$, 0, 3. Signs from left to right: $-$, $+$, $-$, $+$.

Solution: $(-\infty, -5/2] \cup [0, 3]$.

Scoring: 1 point for the factorization and critical numbers; 1 point for the correct intervals.

Question 6 Rational Inequality (3 points)

Subtract 3: $\frac{(x + 4 - 3(x - 1))}{(x - 1)} = \frac{(7 - 2x)}{(x - 1)} < 0$.

Critical numbers 1 and $7/2$. Signs: $-$ for $x < 1$, $+$ on $(1, 7/2)$, $-$ for $x > 7/2$. Both endpoints are excluded.

$(-\infty, 1) \cup (7/2, \infty)$.

Scoring: 1 point for combining into one fraction; 1 point for critical numbers and signs; 1 point for the interval answer.

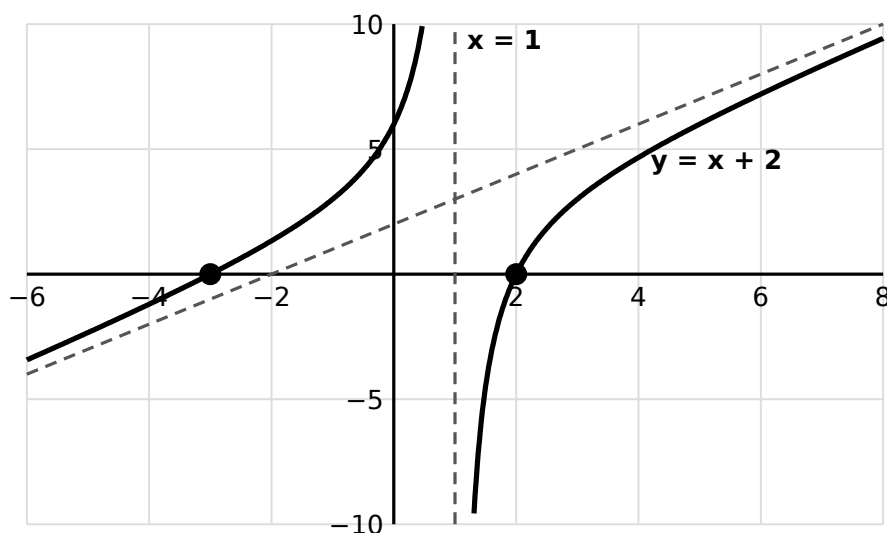
Question 7 Build a Polynomial (2 points)

The conjugate $-3i$ is also a zero, giving the factor $x^2 + 9$. Then $P(x) = (x + 1)^2(x^2 + 9) = (x^2 + 2x + 1)(x^2 + 9)$.

$$x^4 + 2x^3 + 10x^2 + 18x + 9.$$

Scoring: 1 point for the conjugate factor; 1 point for the correct expansion.

Question 8 Analyze a Rational Function (3 points)



Asymptotes and intercepts.

- The denominator is 0 at $x = 1$.
- $x^2 + x - 6 = (x + 2)(x - 1) - 4$, so the quotient is $x + 2$: $y = x + 2$.
- Set the numerator to 0: $(x + 3)(x - 2) = 0$, so **-3 and 2**.

Scoring: 1 point per part.

Question 9 Unit Cost Over Time (2 points)

- $A(n) = 5 + 1200/n$, so the asymptote is $y = 5$: with very many items the average cost approaches \$5.
- $5 + 1200/n \leq 15$ gives $1200 \leq 10n$, so **$n \geq 120$ items**.

Scoring: 1 point per part.