

Polynomial and Rational Functions: Zeros and Asymptotes

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9

Each answer lists the candidate zeros, the division or factoring step and the final result in bold. Use the checks, such as counting zeros against the degree, to confirm your own solutions.

Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · **Level:** Grade 12 Precalculus

Problem 1 List Possible Rational Zeros

The constant term is -10 , with factors $p = \pm 1, \pm 2, \pm 5, \pm 10$. The leading coefficient is 3, with factors $q = \pm 1, \pm 3$.

$\pm 1, \pm 2, \pm 5, \pm 10, \pm 1/3, \pm 2/3, \pm 5/3, \pm 10/3$.

Problem 2 End Behavior from the Leading Term

- (a) Odd degree, negative coefficient: **rises on the left, falls on the right.**
- (b) Even degree, positive coefficient: **rises on both ends.**
- (c) Even degree, negative coefficient: **falls on both ends.**

Problem 3 Conjugate Zeros

Non-real zeros come in conjugate pairs, so the conjugates must also be zeros: **$2 + 5i$ and $-1 - i$** . That gives four zeros, matching the degree.

Problem 4 Classify the Asymptote Type

- (a) The numerator's degree is 3 and the denominator's is 2, so there is a slant asymptote. Dividing gives $x + \frac{(4x+1)}{(x^2-4)}$, so **$y = x$** .
- (b) Equal degrees: horizontal asymptote **$y = 3$** .
- (c) Denominator degree is larger: horizontal asymptote **$y = 0$** .

Problem 5 Read a Sign Chart

Positive regions are between -5 and 1 , and to the right of 4 . The inequality is strict, so the critical numbers are excluded.

$$(-5, 1) \cup (4, \infty).$$

Problem 6 Count the Complex Zeros

$$\text{Degree} = 3 + 2 + 1 = 6.$$

Zeros: 4 (multiplicity 3), $3i$ and $-3i$ (each multiplicity 1), -1 (multiplicity 1).

Total $3 + 1 + 1 + 1 = \mathbf{6 \text{ zeros}}$, equal to the degree.

Problem 7 Find All Zeros with Candidates

Candidates: $\pm 1, \pm 2, \pm 4, \pm 1/2$. Try 1 : $2 + 7 - 5 - 4 = 0$, so 1 is a zero.

Synthetic division by $(x - 1)$ gives $2x^2 + 9x + 4 = (2x + 1)(x + 4)$.

Zeros: $\mathbf{1, -1/2, -4}$.

Problem 8 Real and Complex Zeros Together

Candidates are the divisors of 30 . Since $P(-3) = -27 + 9 - 12 + 30 = 0$, divide by $(x + 3)$ to get $x^2 - 2x + 10$.

$$\text{Quadratic formula: } x = \frac{(2 \pm \sqrt{4 - 40})}{2} = 1 \pm 3i.$$

Zeros: $\mathbf{-3, 1 + 3i, 1 - 3i}$.

Problem 9 Build a Polynomial from Zeros

The conjugate $3 + 2i$ is also a zero. Its quadratic factor is $(x - 3 + 2i)(x - 3 - 2i) = (x - 3)^2 + 4 = x^2 - 6x + 13$.

Multiply by $(x - 5)$: $\mathbf{x^3 - 11x^2 + 43x - 65}$.

Problem 10 Slant Asymptote by Division

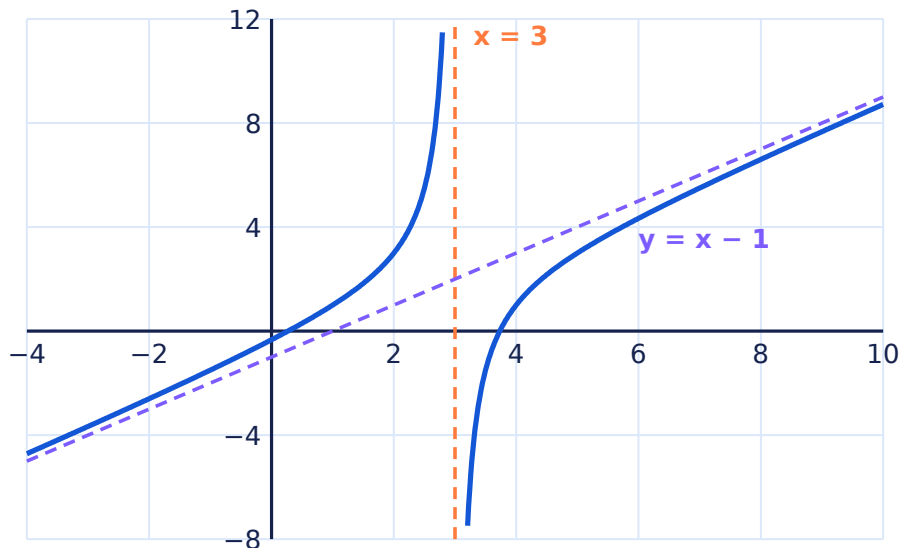
$$\text{Divide: } 3x^2 - x + 4 = (3x + 5)(x - 2) + 14.$$

So $f(x) = 3x + 5 + \frac{14}{(x - 2)}$. Slant asymptote $\mathbf{y = 3x + 5}$; vertical asymptote $\mathbf{x = 2}$.

Problem 11 Identify Asymptotes on a Graph

The vertical dashed line is at $x = 3$. The slant line passes through $(-4, -5)$ and $(10, 9)$, so its slope is $14/14 = 1$ and its equation is $y = x - 1$.

Division: $x^2 - 4x + 1 = (x - 1)(x - 3) - 2$, so the quotient is $x - 1$, which matches.

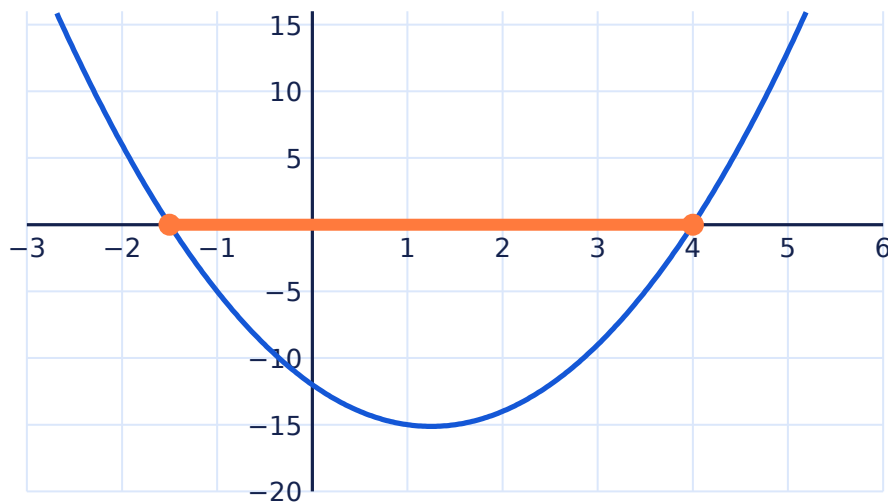


Labeled asymptotes.

Problem 12 Quadratic Inequality

Factor: $(2x + 3)(x - 4) \leq 0$. Critical numbers: $-3/2$ and 4 . The parabola opens upward, so it is at most 0 between them.

$[-3/2, 4]$.



The curve is on or below the axis between the zeros.

Problem 13 Rational Inequality with a Constant

Subtract 2: $\frac{(x+1-2(x-5))}{(x-5)} = \frac{(11-x)}{(x-5)} \leq 0$.

Critical numbers 5 (excluded) and 11. Signs: $-$ for $x < 5$, $+$ on $(5, 11)$, $-$ for $x > 11$. Solution: $(-\infty, 5) \cup [11, \infty)$.



Shaded parts are where the expression is at most 0.

Problem 14 Zeros and Multiplicity from a Graph

Zeros -2 and 3 are crossings (multiplicity 1); 1 is a touch (multiplicity 2). The multiplicities add to 4, the degree.

Leading coefficient negative and even degree: **falls on both ends**.

Equation: $P(x) = -(x+2)(x-1)^2(x-3)$.

Problem 15 Solve a Quartic over the Complex Numbers

Factor like a quadratic in x^2 : $(x^2 - 4)(x^2 + 3) = 0$.

$x^2 = 4$ gives $x = \pm 2$; $x^2 = -3$ gives $x = \pm i\sqrt{3}$.

$x = 2, -2, i\sqrt{3}, -i\sqrt{3}$.

Problem 16 Dimensions of a Crate

$x(x+2)(x-1) = 72$ gives $x^3 + x^2 - 2x - 72 = 0$.

Testing the divisors of 72, $x = 4$ works: $64 + 16 - 8 - 72 = 0$. The quotient is $x^2 + 5x + 18$.

Discriminant: $25 - 72 = -47 < 0$, so the other zeros are non-real. The crate is **4 ft by 6 ft by 3 ft**.

Problem 17 Average Cost and Its Asymptote

Divide each term by n : $A(n) = 2n + 30 + \frac{450}{n}$, so the slant asymptote is $y = 2n + 30$.

Solve $2n^2 + 30n + 450 \leq 110n$ ($n > 0$): $2n^2 - 80n + 450 \leq 0$, so $n^2 - 40n + 225 \leq 0$. Roots: 20

$$\pm 5\sqrt{7} \approx 6.77 \text{ and } 33.23.$$

Check: $A(7) \approx \$108.29$ and $A(33) \approx \$109.64$, but $A(6) = \$117$ and $A(34) \approx \$111.24$. So **7 to 33 kits**.

Problem 18 Ball Above a Height

Solve $-16t^2 + 48t + 64 > 32$, so $-16t^2 + 48t + 32 > 0$ and $t^2 - 3t - 2 < 0$.

Roots: $(3 \pm \sqrt{17})/2 \approx -0.56$ and 3.56 . With $t \geq 0$, the ball is above 32 feet for $0 \leq t < (3 + \sqrt{17})/2 \approx \mathbf{3.56}$ seconds. (It lands at $t = 4$.)

Problem 19 Medicine Concentration Window

Solve $\frac{40t}{(t^2 + 16)} \geq 2.5$. The denominator is always positive, so $40t \geq 2.5t^2 + 40$, which gives $t^2 - 16t + 16 \leq 0$.

Roots: $8 \pm 4\sqrt{3} \approx 1.07$ and 14.93 . The concentration is at least 2.5 mg/L for $\mathbf{8 - 4\sqrt{3} \leq t \leq 8 + 4\sqrt{3}}$ (about **1.07 to 14.93 hours**).

Problem 20 Use a Given Zero to Solve a Quartic

The conjugate $-2i$ is a zero too, giving the factor $(x - 2i)(x + 2i) = x^2 + 4$.

Divide: $P(x) = (x^2 + 4)(x^2 - 2x + 2)$.

The quadratic formula gives $x = \frac{(2 \pm \sqrt{4 - 8})}{2} = 1 \pm i$. Zeros: **$2i, -2i, 1 + i, 1 - i$** .