

Polynomial and Rational Functions: Zeros and Asymptotes

Grade 12 – Precalculus math · Worksheet · Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9

Name: _____ Date: _____ Class: _____

These 20 problems cover the Rational Root Theorem, complex and conjugate zeros, end behavior, slant asymptotes and sign-chart inequalities. They train standards A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d and N-CN.C.9. The last six problems use crates, kit costs, a thrown ball and a medicine dose, so every inequality and asymptote has a real meaning.

Standards: A-APR.B.2, A-APR.B.3, F-IF.C.7c, F-IF.C.7d, N-CN.C.9 · **Level:** Grade 12 Precalculus

Problem 1 List Possible Rational Zeros (easy)

List every possible rational zero of $P(x) = 3x^3 - x^2 + 5x - 10$.

Problem 2 End Behavior from the Leading Term (easy)

Describe the end behavior of each function in words. (a) $f(x) = -2x^5 + 7x - 1$ (b) $g(x) = 4x^4 - x^3$ (c) $h(x) = -x^6 + 9x^2$

Problem 3 Conjugate Zeros (easy)

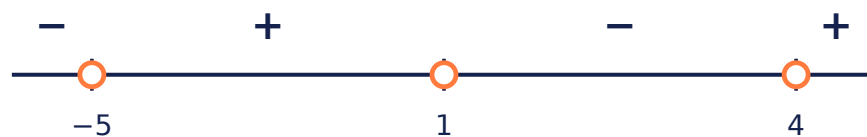
Suppose P has degree 4 and real coefficients, and two of its zeros are $2 - 5i$ and $-1 + i$. Name the remaining two zeros.

Problem 4 Classify the Asymptote Type (easy)

Without graphing, state whether each function has a horizontal asymptote, a slant asymptote or neither, and give its equation. (a) $\frac{x^3 + 1}{x^2 - 4}$ (b) $\frac{3x^2 + 1}{x^2 - 9}$ (c) $\frac{x + 5}{x^2 + 2}$

Problem 5 Read a Sign Chart (easy)

The sign chart shows the sign of $Q(x) = (x + 5)(x - 1)(x - 4)$ on each interval. Solve $Q(x) > 0$.



Signs of $Q(x)$.

Problem 6 Count the Complex Zeros (easy)

Let $P(x) = (x - 4)^3(x^2 + 9)(x + 1)$. Give the degree, list all complex zeros counting repeats, and confirm the Fundamental Theorem of Algebra count.

Problem 7 Find All Zeros with Candidates (medium)

Find all zeros of $P(x) = 2x^3 + 7x^2 - 5x - 4$.

Problem 8 Real and Complex Zeros Together (medium)

Determine every zero, real and non-real, of $P(x) = x^3 + x^2 + 4x + 30$.

Problem 9 Build a Polynomial from Zeros (medium)

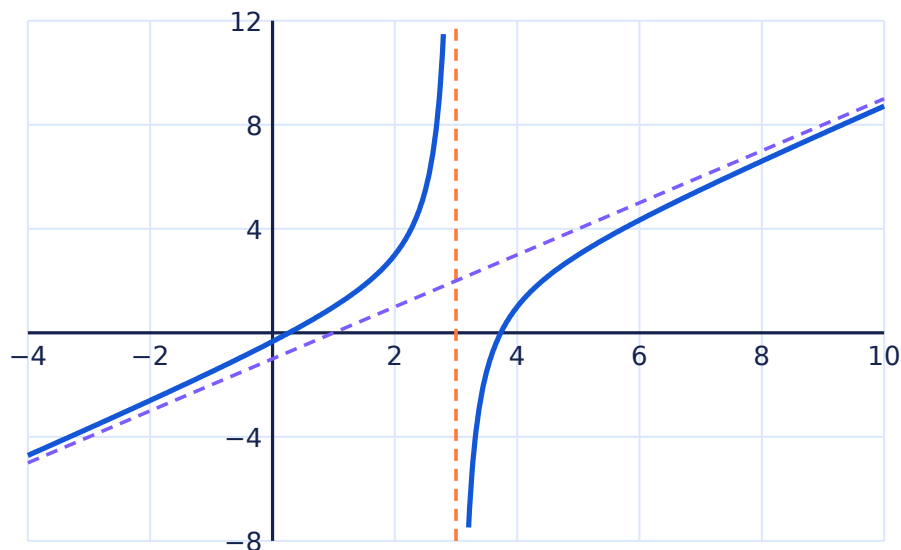
Construct a monic cubic with real coefficients whose zeros include 5 and $3 - 2i$. Multiply out the factors.

Problem 10 Slant Asymptote by Division (medium)

Find the slant asymptote and the vertical asymptote of $f(x) = \frac{(3x^2 - x + 4)}{(x - 2)}$.

Problem 11 Identify Asymptotes on a Graph (medium)

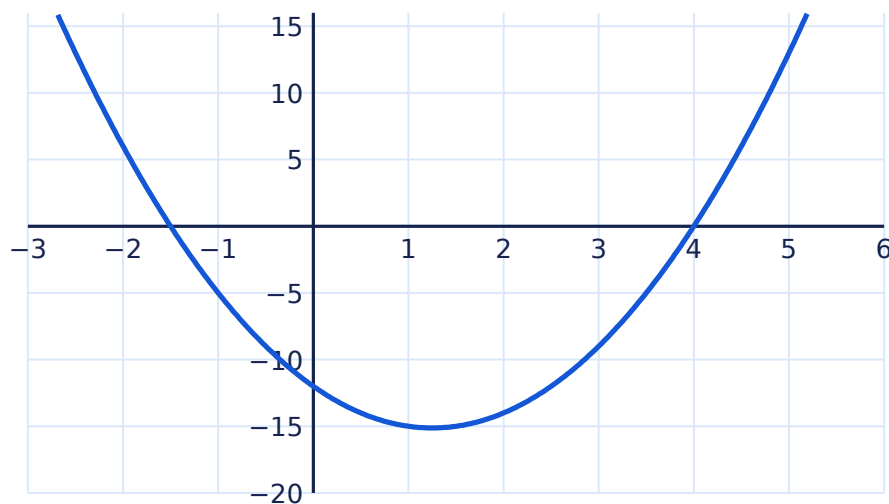
The graph shows $f(x) = \frac{(x^2 - 4x + 1)}{(x - 3)}$ with two dashed asymptotes. Write their equations, then confirm the slant one by division.



Rational function with two dashed asymptotes.

Problem 12 Quadratic Inequality (medium)

The graph of $y = 2x^2 - 5x - 12$ is shown. Solve $2x^2 - 5x - 12 \leq 0$ algebraically and give the solution set as an interval.



Graph of $y = 2x^2 - 5x - 12$.

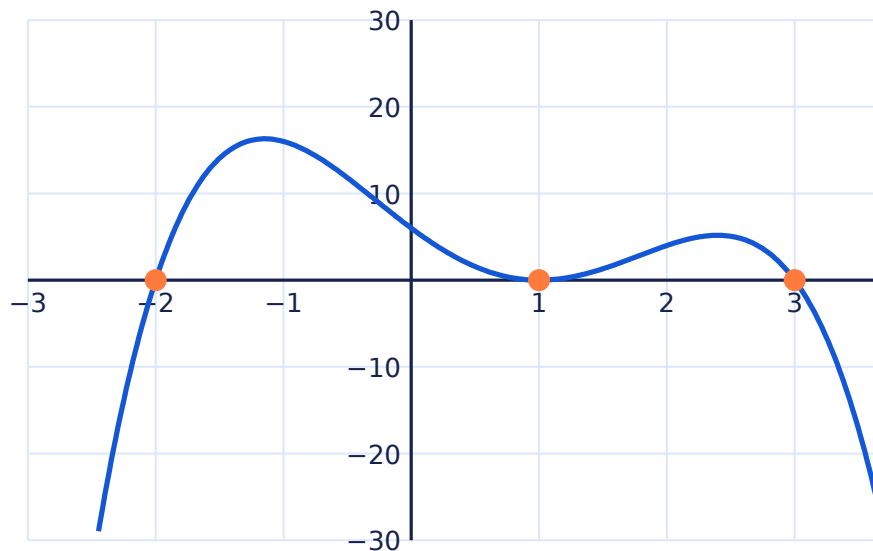
Problem 13 Rational Inequality with a Constant (medium)

Solve $\frac{x+1}{x-5} \leq 2$.

Problem 14 Zeros and Multiplicity from a Graph (medium)

The graph is a polynomial with leading coefficient -1 and degree 4 . It meets the x-axis at -2 , 1 and 3 and touches (does not cross) at 1 . Find each zero's multiplicity, describe the end

behavior, and write a possible equation.



Graph of a degree-4 polynomial.

Problem 15 Solve a Quartic over the Complex Numbers (medium)

Solve $x^4 - x^2 - 12 = 0$ over the complex numbers.

Problem 16 Dimensions of a Crate (hard)

A crate has width x feet, length $x + 2$ feet and height $x - 1$ feet. Its volume is 72 cubic feet. Set up the equation, find the real value of x with the Rational Root Theorem, and show that the other two solutions are not real.

Problem 17 Average Cost and Its Asymptote (hard)

A robotics club builds n kits for a total cost of $2n^2 + 30n + 450$ dollars. The average cost per kit is $A(n) = \frac{(2n^2 + 30n + 450)}{n}$. Find the slant asymptote, then find how many kits (whole numbers) keep the average cost at or below \$110.

Problem 18 Ball Above a Height (hard)

A ball is thrown so that its height is $h(t) = -16t^2 + 48t + 64$ feet after t seconds. For how long is it above 32 feet before it lands? Give the exact interval and a decimal approximation.

Problem 19 Medicine Concentration Window (hard)

The concentration of a medicine in the blood is $C(t) = \frac{40t}{(t^2 + 16)}$ mg/L after t hours. When is the

concentration at least 2.5 mg/L? Give exact endpoints and approximations.

Problem 20 Use a Given Zero to Solve a Quartic (hard)

Given that $2i$ is a zero of $P(x) = x^4 - 2x^3 + 6x^2 - 8x + 8$, find every zero.