

Series, Binomial Expansions and Limits

Grade 12 – Precalculus math · Test Answer Key · Standards: A-APR.C.5, A-SSE.B.4, F-BF.A.2

Use the scoring notes for partial credit. Most items give one point for the setup, such as the first term, the ratio or the coefficients, and one point for the correct result.

Standards: A-APR.C.5, A-SSE.B.4, F-BF.A.2 · **Level:** Grade 12 Precalculus

Question 1 Evaluating a Sum (2 points)

The terms form an arithmetic sequence: $a_1 = 2$, $a_8 = 37$. Sum = $\frac{8}{2}(2 + 37) = 4(39) = 156$.

Answer: **156**

Scoring: 1 point for identifying the first and last terms, 1 point for the sum.

Question 2 Twenty-five Terms (2 points)

$a_{25} = -6 + 24(4) = 90$. $S_{25} = \frac{25}{2}(-6 + 90) = \frac{25}{2}(84) = 1,050$.

Answer: **1,050**

Scoring: 1 point for the 25th term, 1 point for the sum.

Question 3 A Geometric Sum (2 points)

$S_8 = 6 \cdot \frac{(1 - (-2)^8)}{(1 - (-2))} = 6 \cdot \frac{(1 - 256)}{3} = 6 \cdot (-85) = -510$.

Answer: **-510**

Scoring: 1 point for the formula with correct values, 1 point for the result.

Question 4 Infinite Series (3 points)

(a) $r = -12 \div 18 = -\frac{2}{3}$. Since $|r| < 1$, the series converges.

(b) $S = \frac{18}{(1 + 23)} = \frac{18}{53} = \frac{54}{5} = 10.8$.

(c) $0.181818... = 0.18 + 0.0018 + ...$, with $a_1 = 0.18$ and $r = 0.01$, so $S = \frac{0.18}{0.99} = \frac{2}{11}$.

Answer: **(a) $r = -\frac{2}{3}$, converges; (b) $\frac{54}{5}$; (c) $\frac{2}{11}$**

Scoring: 1 point for each part.

Question 5 Expanding a Binomial (2 points)

The coefficients are 1, 4, 6, 4, 1. $(x - 2)^4 = x^4 + 4(-2)x^3 + 6(4)x^2 + 4(-8)x + 16$.

Answer: $x^4 - 8x^3 + 24x^2 - 32x + 16$

Scoring: 1 point for correct coefficients and powers, 1 point for correct signs and constants.

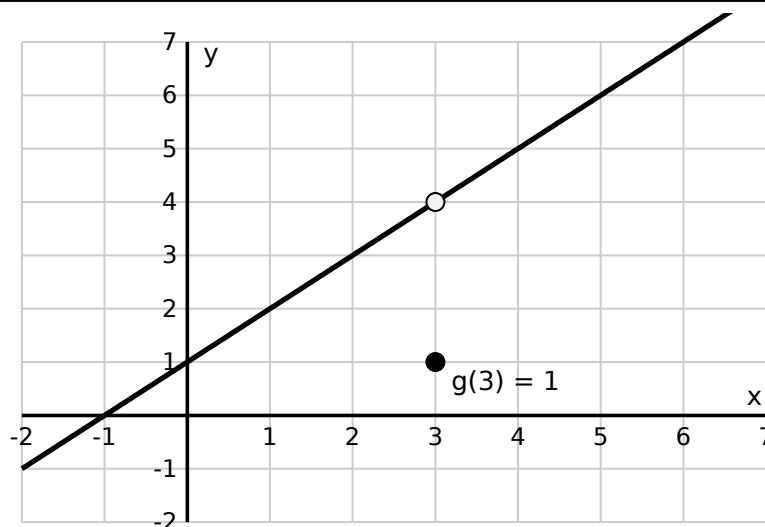
Question 6 A Term of an Expansion (2 points)

The fourth term has $k = 3$: $C(5, 3)(3x)^2(-y)^3 = 10 \cdot 9x^2 \cdot (-y^3)$.

Answer: $-90x^2y^3$

Scoring: 1 point for the setup with $C(5, 3)$, 1 point for the term.

Question 7 Reading a Graph (3 points)



The outputs approach 4, but $g(3) = 1$.

(a) As x approaches 3 from either side, $g(x)$ approaches $3 + 1 = 4$.

(b) The filled dot is at $(3, 1)$, so $g(3) = 1$.

(c) No. The limit (4) does not equal the function value (1), so the graph has a jump at $x = 3$.

Answer: **(a) 4; (b) 1; (c) not continuous**

Scoring: 1 point for each part.

Question 8 Two Limits (2 points)

(a) $x^2 + x - 2 = (x + 2)(x - 1)$, so for $x \neq -2$ the expression is $x - 1$, which approaches -3 .

(b) Divide the top and bottom by n : $\frac{(4 + 7/n)}{(1 + 1/n)} \rightarrow \frac{4}{1} = 4$.

Answer: **(a) -3 ; (b) 4**

Scoring: 1 point for each part.

Question 9 Bouncing Ball (2 points)

The first drop is 8 ft. The rebound heights 6, 4.5, ... form a geometric series with $a_1 = 6$ and $r = 0.75$, and each rebound is traveled twice.

Up-and-down distance = $2 \cdot \frac{6}{(1 - 0.75)} = 2(24) = 48$. Total = $8 + 48 = 56$ ft.

Answer: **56 feet**

Scoring: 1 point for the series setup, 1 point for the total.