

Series, Binomial Expansions and Limits

Grade 12 – Precalculus math · Worksheet · Standards: A-APR.C.5, A-SSE.B.4, F-BF.A.2

Name: _____ Date: _____ Class: _____

These problems cover sigma notation, arithmetic and geometric series, infinite geometric series, the binomial theorem with Pascal's triangle, and informal limits of sequences and functions (A-APR.C.5, A-SSE.B.4 and F-BF.A.2).

The last problems use theater seating, savings, a bouncing ball and medicine doses.

Standards: A-APR.C.5, A-SSE.B.4, F-BF.A.2 · **Level:** Grade 12 Precalculus

Problem 1 Evaluating a Sum (easy)

Evaluate $\sum_{k=1}^6 (k^2 + 1)$.

Problem 2 Writing Sigma Notation (easy)

Write $4 + 7 + 10 + 13 + 16$ in sigma notation.

Problem 3 An Arithmetic Series (easy)

Find the sum $7 + 11 + 15 + \dots + 51$.

Problem 4 A Geometric Series (easy)

Add up the first seven terms of the geometric sequence that begins 4, 12, 36, and so on. Use the sum formula.

Problem 5 An Infinite Sum (easy)



Partial sums of the series.

The chart shows the partial sums of $9 + 6 + 4 + \frac{8}{3} + \dots$. Find the sum of the series.

Problem 6 Divergent or Not? (easy)

Does $8 + 9.6 + 11.52 + \dots$ have a sum? Explain.

Problem 7 How Many Terms? (medium)

An arithmetic series has first term -4 and common difference 3 . Its sum is 217 . Find the number of terms.

Problem 8 An Alternating Sum (medium)

Evaluate $\sum_{k=1}^7 5(-2)^{k-1}$.

Problem 9 A Repeating Decimal (medium)

Write $0.272727\dots$ as an infinite geometric series and find the fraction it equals.

Problem 10 An Alternating Infinite Series (medium)

Find $\sum_{k=1}^{\infty} 6(-\frac{1}{2})^{k-1}$.

Problem 11 Pascal's Triangle Expansion (medium)

row 0						1					
row 1					1		1				
row 2				1		2		1			
row 3			1		3		3		1		
row 4		1		4		6		4		1	
row 5	1		5		10		10		5		1

Pascal triangle, rows 0 to 5.

Use row 5 of Pascal's triangle to expand $(x + 2)^5$.

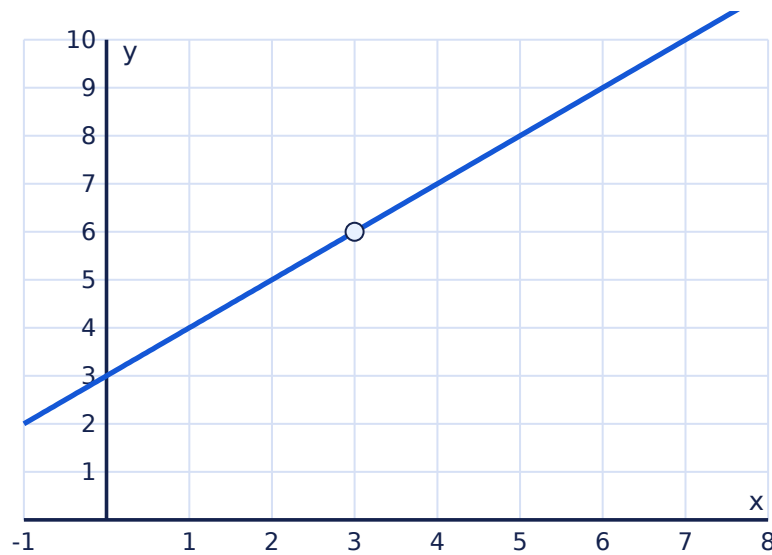
Problem 12 Expanding with a Coefficient (medium)

Expand $(2x - 3)^4$.

Problem 13 A Single Coefficient (medium)

Find the coefficient of x^3 in the expansion of $(x + 4)^7$.

Problem 14 A Limit from a Table (medium)



The graph of $f(x) = (x^2 - 9)/(x - 3)$.

Let $f(x) = \frac{(x^2 - 9)}{(x - 3)}$. (a) Complete the table of values. (b) Find $\lim_{x \rightarrow 3} f(x)$ and explain why $f(3)$ does not exist.

x	2.9	2.99	2.999	3.001	3.01
$f(x)$	?	?	?	?	?

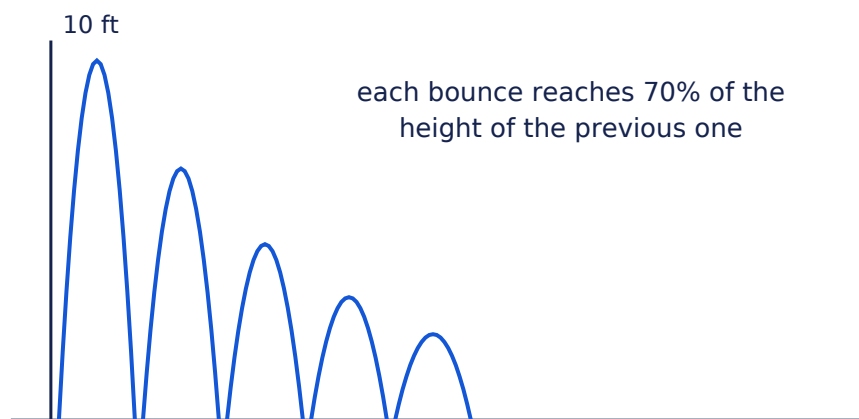
Problem 15 Theater Seating (hard)

A theater has 18 rows. The first row has 14 seats and each row after it has 2 more seats than the row before. (a) How many seats are in row 18? (b) How many seats are there in all?

Problem 16 Monthly Savings Plan (hard)

A student deposits \$40 in month 1 and increases each month's deposit by 5% over the previous month, for 24 months. Find the total deposited, to the nearest cent.

Problem 17 Bouncing Ball (hard)



A bouncing ball: each rebound is 70% as high as the last.

A ball is dropped from 10 feet. After each bounce it rises to 70% of the height of the previous fall. Find the total distance it travels before it comes to rest, to the nearest hundredth.

Problem 18 Medicine Dosage (hard)

A patient takes a 200 mg dose each morning. By the next morning, 30% of the drug from the previous day is still in the body. (a) Find the amount in the body right after the first, second and third doses. (b) Find the long-run amount right after a dose.

Problem 19 A Special Term (hard)

In the expansion of $(x^2 + \frac{1}{x})^6$, find (a) the constant term and (b) the coefficient of x^3 .

Problem 20 Limits of a Function and a Sequence (hard)

Find (a) $\lim_{x \rightarrow -1} \frac{(x^3 + 1)}{(x + 1)}$ and (b) $\lim_{n \rightarrow \infty} \frac{(6n^2 + 5)}{(2n^2 - n + 3)}$. (c) Explain why part (a) can have a limit even though the expression is undefined at $x = -1$.