

Graphing Sine, Cosine and Tangent Functions

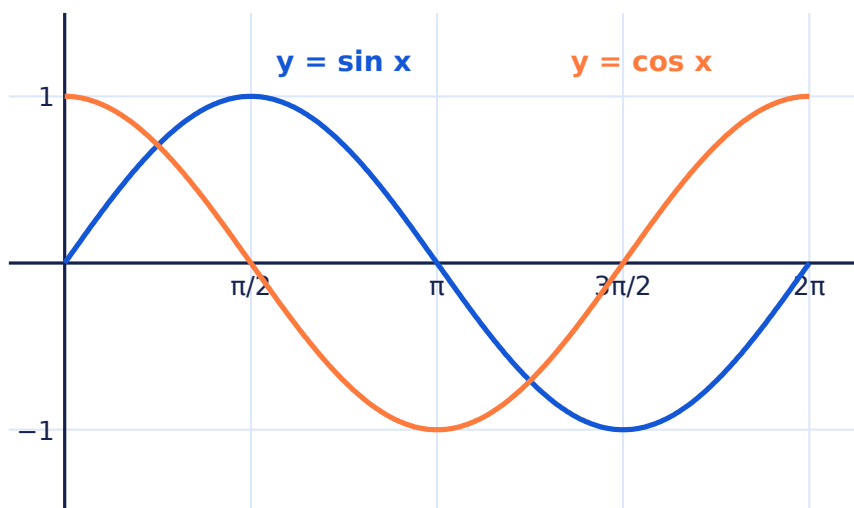
Grade 12 – Precalculus math · Lesson · Standards: F-TF.B.5, F-IF.C.7e, F-BF.B.3

Quick answer: Sine and cosine graphs are waves with amplitude 1, period 2π and midline $y = 0$. In $y = A \sin(B(x - C)) + D$, A sets the amplitude, B sets the period $2\pi/|B|$, C shifts the wave horizontally, and D moves the midline. Tangent has period π and vertical asymptotes at $\pi/2 + k\pi$.

Standards: F-TF.B.5, F-IF.C.7e, F-BF.B.3 · **Level:** Grade 12 Precalculus

What do the graphs of sine and cosine look like?

Both graphs are smooth waves that repeat every 2π , with a highest value of 1 and a lowest value of -1 . Sine starts at the midline and rises; cosine starts at its maximum. Shifting cosine right by $\pi/2$ gives sine.



Sine and cosine: amplitude 1, period 2π , midline $y = 0$.

How do amplitude, period, shift and midline change the wave?

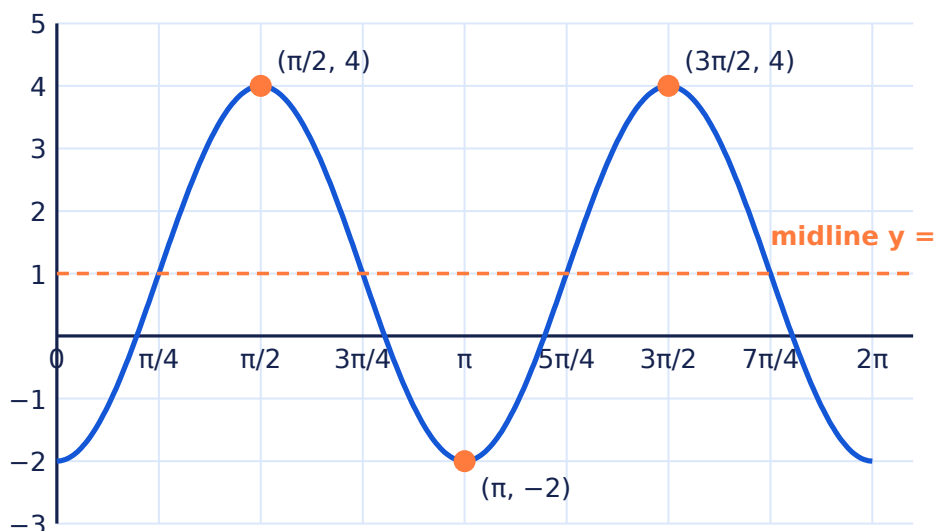
General sinusoid

$$y = A \sin(B(x - C)) + D$$

- Amplitude $|A|$: distance from the midline to a peak.
- Period $2\pi/|B|$: the length of one full wave.
- Phase shift C : right if positive, left if negative.
- Midline $y = D$: maximum is $D + |A|$, minimum $D - |A|$.

Worked example

For $y = 3 \sin(2(x - \pi/4)) + 1$: amplitude **3**, period $2\pi/2 = \pi$, shift **$\pi/4$ right**, midline **$y = 1$** . The maximum is 4 and the minimum is -2 .



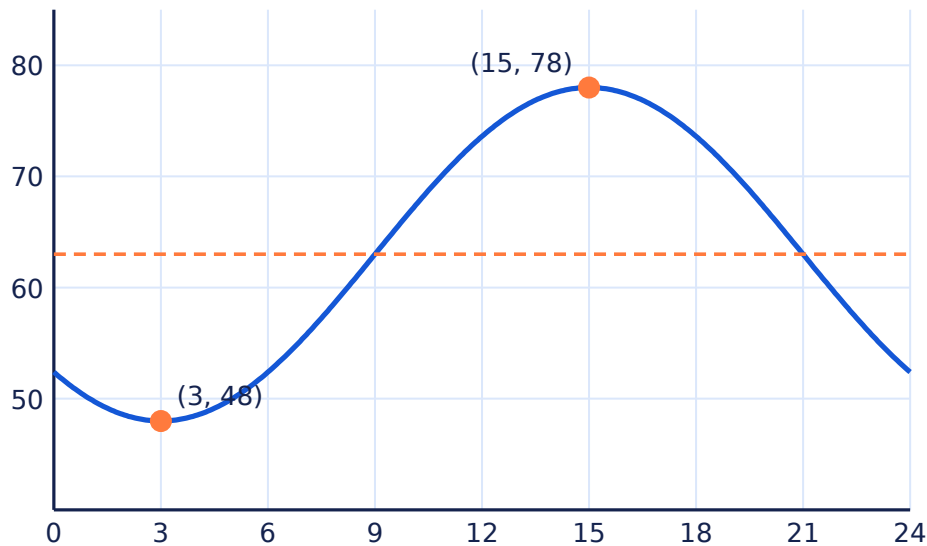
Amplitude 3, period π , shift $\pi/4$ right, midline $y = 1$.

How do you write a sinusoidal model from data?

Method: model from a maximum and a minimum

1. Amplitude $A = (\max - \min)/2$, midline $D = (\max + \min)/2$.
2. The distance from a maximum to the next minimum is half a period, so find the period and use $B = 2\pi/\text{period}$.
3. Use cosine if you know where a maximum is: shift C equals that x -value.

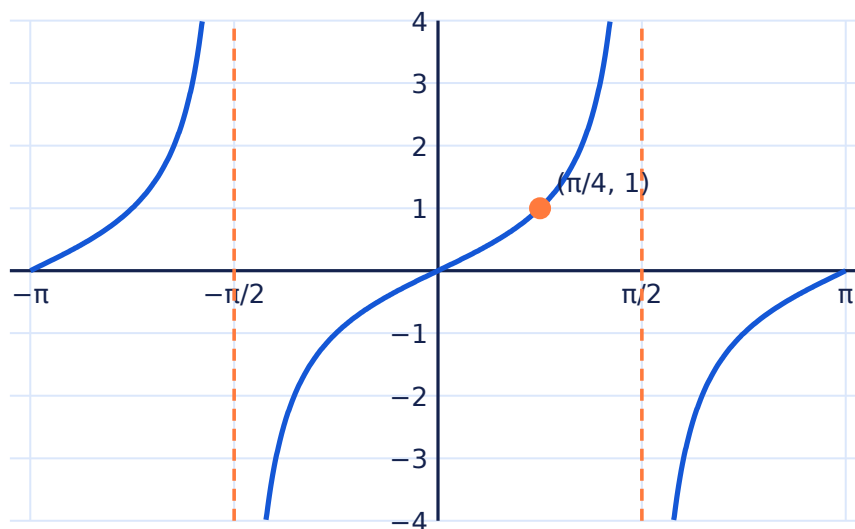
A day's temperature peaks at 78°F at hour 15 and bottoms out at 48°F at hour 3. Then $A = 15$, $D = 63$, the period is 24, so $B = \pi/12$ and **$T(t) = 15\cos(\pi/12(t - 15)) + 63$** .



Temperature over one day: $T(t) = 15\cos(\pi/12 (t - 15)) + 63$.

What does the tangent graph look like?

Tangent has period π , zeros at multiples of π , and vertical asymptotes at $x = \pi/2 + k\pi$, where cosine is 0. It has no amplitude because it is unbounded. For $y = \tan(Bx)$ the period is $\pi/|B|$.



Tangent has period π and asymptotes at odd multiples of $\pi/2$.

Common mistake

Reading the shift from $\sin(2x - \pi)$ as π . Factor first: $\sin(2(x - \pi/2))$ shifts the graph $\pi/2$, not π .

Mathbot's tip

Always factor out B before reading the phase shift. Then plot the midline, the max and min, and connect them with a smooth wave.

Key points to remember

- Amplitude = $|A|$; maximum = $D + |A|$; minimum = $D - |A|$.
- Period of sine and cosine = $2\pi/|B|$; period of tangent = $\pi/|B|$.
- Phase shift is C in $A \sin(B(x - C))$: factor B out first.
- To model data: $A = (\max - \min)/2$, $D = (\max + \min)/2$, $B = 2\pi/\text{period}$.
- Tangent has vertical asymptotes at $x = \pi/2 + k\pi$ and zeros at $x = k\pi$.

Graphing Sine, Cosine and Tangent Functions: frequently asked questions

What are amplitude, period and midline?

Amplitude is the distance from the midline to a peak, period is the horizontal length of one full cycle, and the midline is the horizontal line halfway between the maximum and minimum values.

How do you find the phase shift of a sine or cosine function?

Factor the coefficient of x out of the argument so it reads $B(x - C)$. The shift is C . For $\sin(2x - \pi) = \sin(2(x - \pi/2))$, the shift is $\pi/2$ to the right.

Why does the tangent graph have vertical asymptotes?

Tangent equals sine divided by cosine. At odd multiples of $\pi/2$ the cosine is 0 while the sine is not, so the quotient grows without bound and the graph approaches a vertical line.

How do you write a sinusoidal model from data?

Take the highest and lowest values to get amplitude and midline, measure the time between peaks for the period, and choose cosine with a shift at a maximum or sine with a shift at a midline crossing.

How are the graphs of sine and cosine related?

They have the same shape. Shifting the cosine graph $\pi/2$ units to the right produces the sine graph, so $\sin x = \cos(x - \pi/2)$.