

Graphing Sine, Cosine and Tangent Functions

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-TF.B.5, F-IF.C.7e, F-BF.B.3

Each answer finds the amplitude, midline and period first, then builds or checks the equation and gives the final result in bold. Test your own equations by plugging in a maximum or a midline crossing.

Standards: F-TF.B.5, F-IF.C.7e, F-BF.B.3 · **Level:** Grade 12 Precalculus

Problem 1 Amplitude and Period

- (a) Amplitude **3**, period **2π** .
- (b) Amplitude **2** (the sign only reflects), period $2\pi/4 = \pi/2$.
- (c) Amplitude **$1/2$** , period $2\pi \div (1/3) = \mathbf{6\pi}$.
- (d) Amplitude **5**, period $2\pi/\pi = \mathbf{2}$.

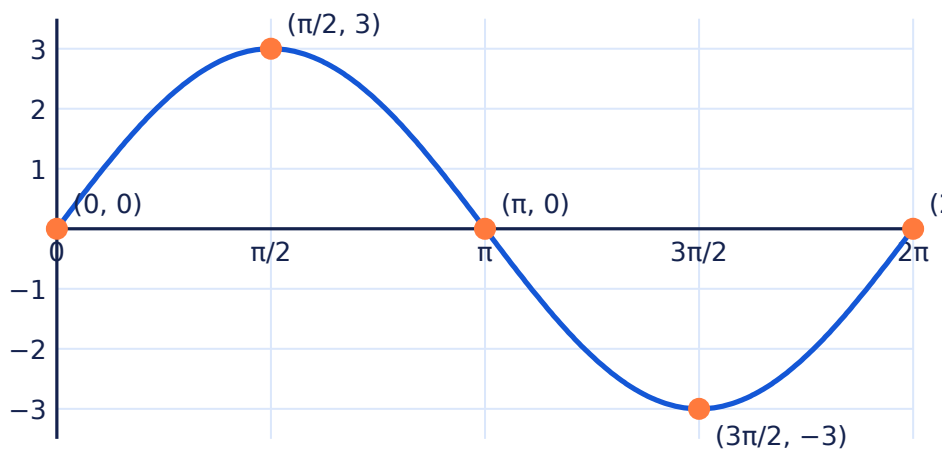
Problem 2 Midline and Range

- (a) Midline **$y = 3$** ; maximum $3 + 2 = \mathbf{5}$; minimum $3 - 2 = \mathbf{1}$.
- (b) Midline **$y = -2$** ; amplitude 1, so the maximum is **-1** and the minimum is **-3** .

Problem 3 Key Points of a Sine Wave

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	3	0	-3	0

Sine is 0, 1, 0, -1 , 0 at these inputs; multiplying by 3 gives **0, 3, 0, -3 , 0**.



The key points of $y = 3 \sin x$.

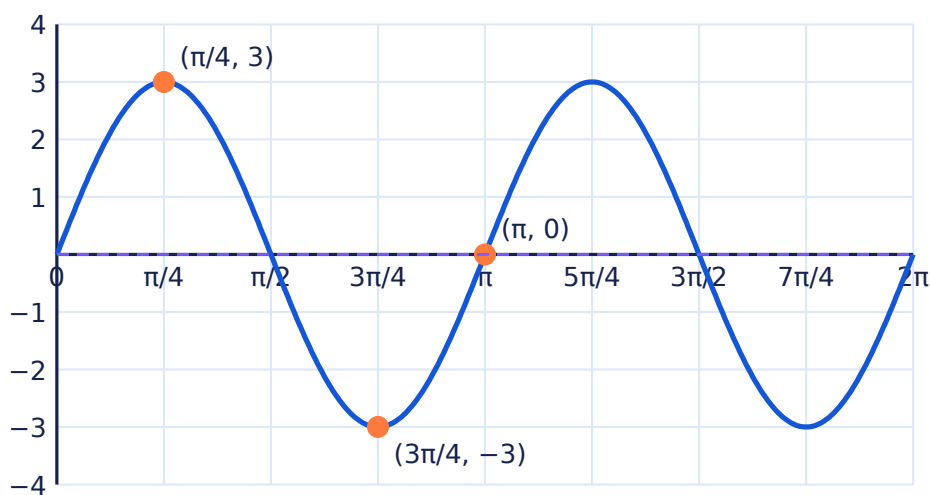
Problem 4 Read the Phase Shift

- (a) $\pi/3$ to the right.
- (b) $\pi/2$ to the left.
- (c) Factor: $\sin(2(x - \pi/2))$, so $\pi/2$ to the right (not π).

Problem 5 Write an Equation from a Graph

The peak is 3, so $A = 3$. The curve starts at 0 and reaches its first peak at $\pi/4$, a quarter of a period, so the period is $4 \cdot \pi/4 = \pi$. Then $B = 2\pi/\pi = 2$.

$y = 3 \sin(2x)$. Check: at $x = \pi/4$, $3 \sin(\pi/2) = 3 \checkmark$.



Labeled key points and midline.

Problem 6 Tangent Properties

Asymptotes where cosine is 0: $x = -\pi/2$ and $x = \pi/2$.

Zeros where sine is 0: $x = -\pi, 0, \pi$.

$$\tan(\pi/4) = 1.$$

Problem 7 Write a Cosine from a Description

$B = 2\pi/6 = \pi/3$. With a maximum at $x = 0$ there is no shift.

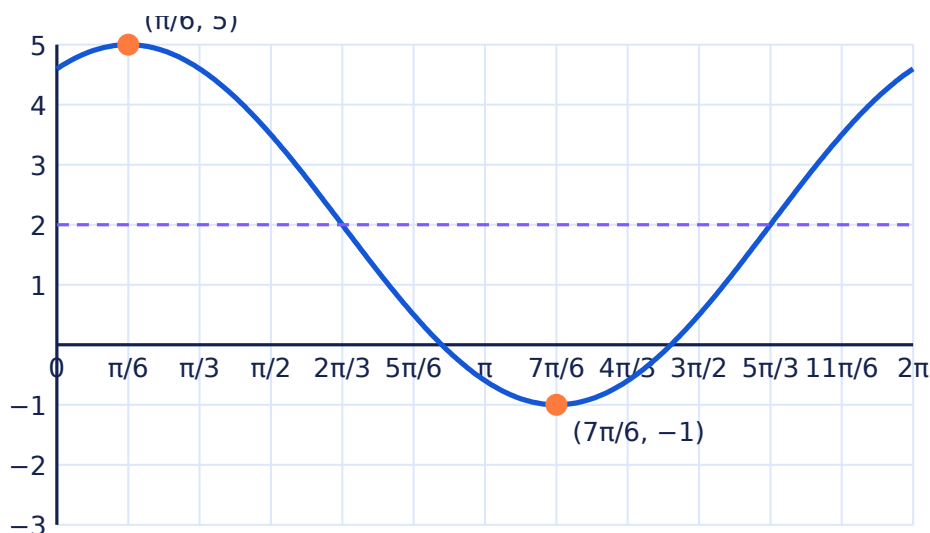
$$y = 2.5 \cos(\pi x/3) - 1. \text{ Check: maximum } 2.5 - 1 = 1.5 \text{ at } x = 0.$$

Problem 8 Equation from a Maximum and a Minimum

$$A = (5 - (-1))/2 = 3; D = (5 + (-1))/2 = 2.$$

Max to min is half a period: $7\pi/6 - \pi/6 = \pi$, so the period is 2π and $B = 1$. The shift is the maximum's location, $\pi/6$.

$$y = 3 \cos(x - \pi/6) + 2.$$



The midline $y = 2$ lies halfway between 5 and -1 .

Problem 9 Reflect, Shift and Compress

$$\text{Factor: } 3x - \pi = 3(x - \pi/3), \text{ so } y = -2 \sin(3(x - \pi/3)) + 4.$$

Amplitude **2** (reflected over the midline), period $2\pi/3 = 2\pi/3$, shift **$\pi/3$ to the right**, midline **$y = 4$** , maximum **6**, minimum **2**.

Problem 10 Tangent Transformations

(a) Period $\pi \div (1/6) = 6\pi$. Asymptotes where $x/6 = \pm\pi/2$: $x = \pm 3\pi$.

(b) Period π . Asymptotes where $x + \pi/6 = \pm\pi/2$: $x = \pi/3$ and $x = -2\pi/3$. The curve passes through $(-\pi/6, -1)$.

Problem 11 Pier Water Depth

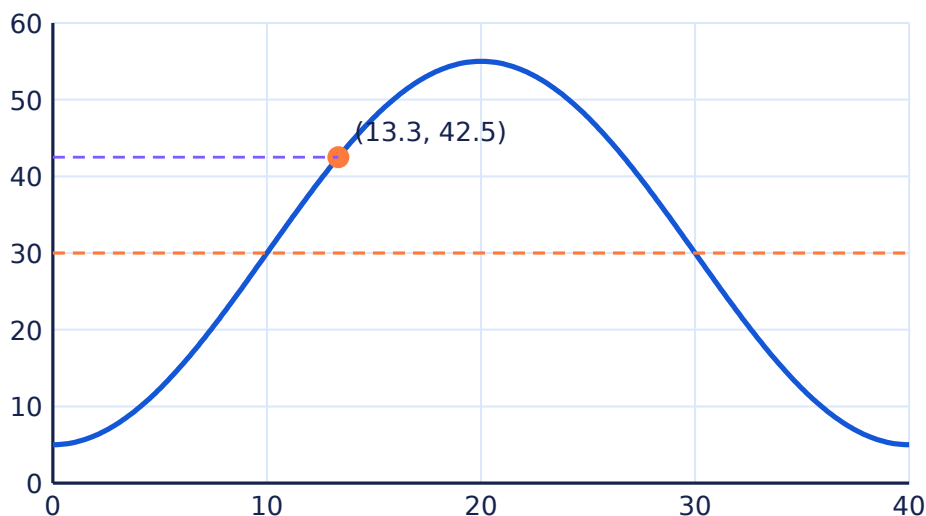
Period $2\pi \div (\pi/6) = 12$ hours. Greatest depth $9 + 4 = 13$ ft; least depth $9 - 4 = 5$ ft.

$$d(2) = 4 \sin(\pi/3) + 9 = 4(0.866) + 9 \approx 12.5 \text{ ft.}$$

Problem 12 Ferris Wheel Height

$$H(0) = 30 - 25 = 5 \text{ ft; } H(10) = 30 - 25 \cos(\pi/2) = 30 \text{ ft; } H(20) = 30 + 25 = 55 \text{ ft.}$$

Solve $30 - 25\cos(\pi t/20) = 42.5$: $\cos(\pi t/20) = -1/2$, so $\pi t/20 = 2\pi/3$ and $t = 40/3 \approx 13.3$ seconds.



The height 42.5 ft is first reached at $t \approx 13.3$.

Problem 13 Model from a Table

Maximum 9, minimum 1: amplitude **4**, midline **$y = 5$** . The period is 8 (minimum at 0 and again at 8), so $B = \pi/4$.

The curve starts at its minimum, so use a reflected cosine: **$y = -4\cos(\pi x/4) + 5$** . Check: $x = 4$ gives $-4(-1) + 5 = 9$ ✓.

Problem 14 Graph of a Compressed Tangent

Period $\pi/5$. The asymptotes shown are $x = \pm\pi/10$, where $5x = \pm\pi/2$.

Zeros where $5x = 0$ or $\pm\pi$: $x = 0$ and, at the ends of the interval, $x = \pm\pi/5$.

At $x = \pi/20$: $\tan(5\pi/20) = \tan(\pi/4) = 1$.

Problem 15 Describe the Transformations

(1) Stretch vertically by a factor of 2 and reflect over the x -axis. (2) Stretch horizontally by a factor of 2, making the period $2\pi \div (1/2) = 4\pi$. (3) Shift up 3.

Amplitude **2**; range **[1, 5]**.

Problem 16 Pedal Height

Lowest $14 - 8 = 6$ in; highest $14 + 8 = 22$ in.

$h(0.3) = 14 - 8\cos(\pi/2) = 14$ in.

Solve $14 - 8\cos(5\pi t/3) = 18$: $\cos(5\pi t/3) = -1/2$, so $5\pi t/3 = 2\pi/3$ and $t = 0.4$ s.

Problem 17 Hours of Daylight

$A = (15.4 - 8.6)/2 = 3.4$, midline $(15.4 + 8.6)/2 = 12$.

$D(80) = 3.4\cos(2\pi(-92)/365) + 12 \approx 11.96$ hours.

Solve $3.4\cos(\cdot) + 12 \geq 14$: $\cos(\cdot) \geq 0.588$, so $|2\pi(d - 172)/365| \leq 0.942$, giving $|d - 172| \leq 54.7$. That is days **118 to 226**, which is **109 days**.

Problem 18 Mass on a Spring

Amplitude **6 cm**; period $2\pi/4\pi = 0.5$ s; frequency $1/0.5 = 2$ cycles per second.

Solve $6 \sin(4\pi t) = 3$: $\sin(4\pi t) = 1/2$, so $4\pi t = \pi/6$ or $5\pi/6$, giving $t = 1/24$ s and $t = 5/24$ s.

Problem 19 Household Voltage

Peak **170 V**; period $2\pi/120\pi = 1/60$ s.

$V(1/240) = 170 \sin(\pi/2) = 170$ V.

$170 \sin(120\pi t) = 85$ gives $\sin(120\pi t) = 1/2$, so $120\pi t = \pi/6$ or $5\pi/6$: $t = \mathbf{1/720 \text{ s}}$ and $\mathbf{1/144 \text{ s}}$.

Problem 20 Model the Tide from Data

Half a period is 6.2 hours, so the period is 12.4 and $B = 2\pi/12.4 \approx 0.5067$. $A = (7.8 - 0.6)/2 = 3.6$ and $D = (7.8 + 0.6)/2 = 4.2$.

$$\mathbf{h(t) = 3.6\cos(0.5067(t - 3)) + 4.2.}$$

$$\mathbf{h(6) \approx 4.4 \text{ ft}; h(12) \approx 3.7 \text{ ft.}}$$