

Proving and Using Trigonometric Identities

Grade 12 – Precalculus math · Test Answer Key · Standards: F-TF.C.8, F-TF.C.9

Each solution shows the work with a scoring note. Award points for a valid first step, for using the correct identity, and for reaching the target, so a late slip costs only one point.

Standards: F-TF.C.8, F-TF.C.9 · **Level:** Grade 12 Precalculus

Question 1 Simplify Expressions (2 points)

(a) $1 - \cos^2 x = \sin^2 x$.

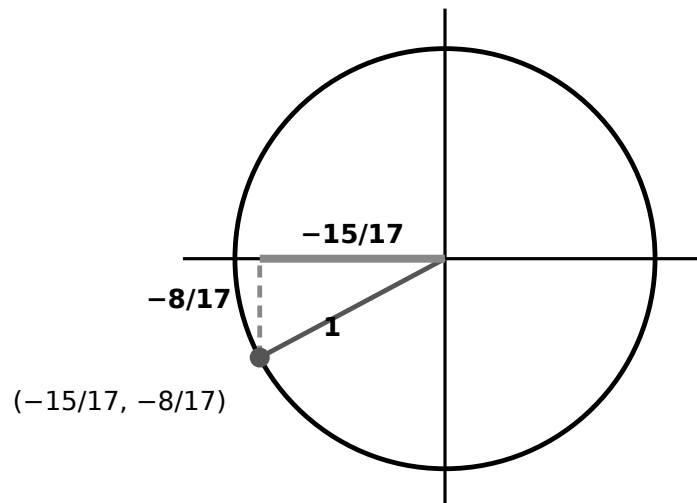
(b) $\sec^2 x \cdot \cos^2 x = 1$.

Scoring: 1 point per part.

Question 2 Find Cosine and Tangent (2 points)

$\cos^2 \theta = 1 - 64/289 = 225/289$. In Quadrant III cosine is negative: $\cos \theta = -15/17$.

$\tan \theta = (-8/17) \div (-15/17) = 8/15$.



A point in Quadrant III.

Scoring: 1 point per value.

Question 3 Verify an Identity (3 points)

Multiply the numerator and denominator of the left side by $(1 + \cos x)$: $\sin x(1 + \cos x)/(1 - \cos^2 x)$.

Since $1 - \cos^2 x = \sin^2 x$, this equals $\sin x(1 + \cos x)/\sin^2 x = \mathbf{(1 + \cos x)/\sin x}$. ■

Scoring: 1 point for a valid first step on one side; 1 point for using the Pythagorean identity; 1 point for reaching the other side.

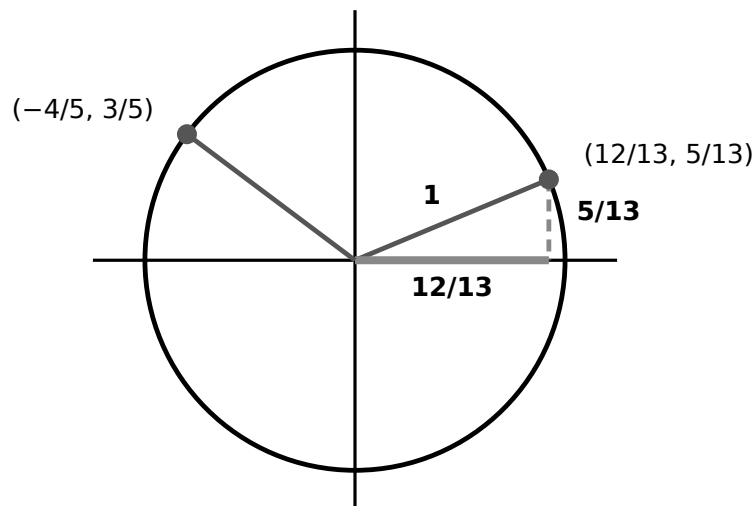
Question 4 Exact Values from Sum Formulas (3 points)

(a) $\cos(60^\circ + 45^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ = (1/2)(\sqrt{2}/2) - (\sqrt{3}/2)(\sqrt{2}/2) = \mathbf{(\sqrt{2} - \sqrt{6})/4}$.

(b) $\tan(\pi/3 - \pi/4) = (\sqrt{3} - 1)/(1 + \sqrt{3}) = (\sqrt{3} - 1)^2/(3 - 1) = (4 - 2\sqrt{3})/2 = \mathbf{2 - \sqrt{3}}$.

Scoring: (a) 1 point for the correct formula and values, 1 point for the simplified answer. (b) 1 point.

Question 5 Sum of Two Angles from a Graph (3 points)



Points P (angle α) and Q (angle β).

- a. $\cos \alpha = 12/13$, $\sin \alpha = 5/13$; $\cos \beta = -4/5$, $\sin \beta = 3/5$.
- b. $\sin(\alpha + \beta) = (5/13)(-4/5) + (12/13)(3/5) = (-20 + 36)/65 = \mathbf{16/65}$.
- c. $\cos(\alpha + \beta) = (12/13)(-4/5) - (5/13)(3/5) = (-48 - 15)/65 = \mathbf{-63/65}$.

Scoring: 1 point per part.

Question 6 Double-Angle Values (2 points)

$$\cos \theta = -3/5.$$

$$\sin 2\theta = 2(4/5)(-3/5) = \mathbf{-24/25}.$$

$$\cos 2\theta = 9/25 - 16/25 = \mathbf{-7/25}.$$

Scoring: 1 point per value.

Question 7 Verify with Double Angles (3 points)

Use $\sin 2x = 2 \sin x \cos x$ and $1 + \cos 2x = 1 + 2\cos^2 x - 1 = 2\cos^2 x$.

The fraction is $2 \sin x \cos x / (2\cos^2 x) = \sin x / \cos x = \mathbf{\tan x}$. ■

Scoring: 1 point for the numerator; 1 point for the denominator; 1 point for simplifying to $\tan x$.

Question 8 Solve a Double-Angle Equation (2 points)

$1 - 2\sin^2 x = \sin x$, so $2\sin^2 x + \sin x - 1 = 0$ and $(2 \sin x - 1)(\sin x + 1) = 0$.

$\sin x = 1/2$ gives $\pi/6, 5\pi/6$; $\sin x = -1$ gives $3\pi/2$. **$x = \pi/6, 5\pi/6, 3\pi/2$.**

Scoring: 1 point for the quadratic in sine; 1 point for all three solutions.