

Proving and Using Trigonometric Identities

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-TF.C.8, F-TF.C.9

Each answer transforms one side only and ends with the result in bold, with the proof symbol after a verified identity. Check your own work with a quick numerical test at a special angle.

Standards: F-TF.C.8, F-TF.C.9 · **Level:** Grade 12 Precalculus

Problem 1 Reciprocal and Quotient Values

$$\tan \theta = (5/13) \div (12/13) = \mathbf{5/12}. \quad \csc \theta = 1 \div (5/13) = \mathbf{13/5}. \quad \sec \theta = \mathbf{13/12}. \quad \cot \theta = \mathbf{12/5}.$$

Problem 2 Simplify with Pythagorean Identities

$$(a) \ 5(1 - \sin^2 x) = \mathbf{5\cos^2 x}. \quad (b) \ 3(\sec^2 x - 1) = \mathbf{3\tan^2 x}. \quad (c) \ \mathbf{1}. \quad (d) \ \cos x \cdot \sin x / \cos x = \mathbf{\sin x}.$$

Problem 3 Find Sine and Tangent from Cosine

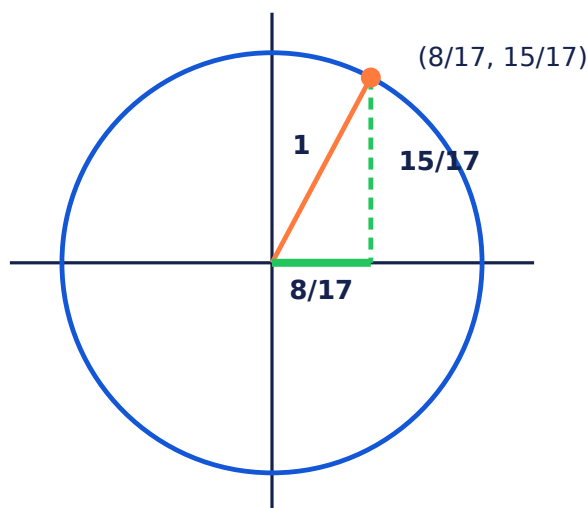
$$\sin^2 \theta = 1 - 25/169 = 144/169. \text{ In Quadrant IV sine is negative: } \sin \theta = \mathbf{-12/13}.$$

$$\tan \theta = (-12/13) \div (5/13) = \mathbf{-12/5}.$$

Problem 4 Check a Point on the Unit Circle

$$(8/17)^2 + (15/17)^2 = 64/289 + 225/289 = 289/289 = 1, \text{ so the point is on the circle.}$$

$$\cos \theta = \mathbf{8/17}, \quad \sin \theta = \mathbf{15/17}, \quad \tan \theta = \mathbf{15/8}, \quad \sec \theta = \mathbf{17/8}.$$



The point $(8/17, 15/17)$ on the unit circle.

Problem 5 Identity or Not?

(a) $(1/2)^2 + (\sqrt{3}/2)^2 = 1/4 + 3/4 = 1$.

(b) $\sin(\pi/3) = \sqrt{3}/2 \approx 0.866$, but $\sin(\pi/6) + \sin(\pi/6) = 1$. Since $0.866 \neq 1$, **it is not an identity**.

Problem 6 Exact Value with a Difference Formula

$$\begin{aligned} \cos(45^\circ - 30^\circ) &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = (\sqrt{2}/2)(\sqrt{3}/2) + (\sqrt{2}/2)(1/2). \\ &= (\sqrt{6} + \sqrt{2})/4 \approx 0.966. \end{aligned}$$

Problem 7 Verify a Quotient Identity

Left side: $1/\cos x - \cos x = (1 - \cos^2 x)/\cos x = \sin^2 x/\cos x = \sin x \cdot (\sin x/\cos x) = \sin x \tan x$.
■

Problem 8 Verify with Reciprocals

Left side: $1/\sin x - \sin x = (1 - \sin^2 x)/\sin x = \cos^2 x/\sin x = \cos x \cdot (\cos x/\sin x) = \cos x \cot x$.
■

Problem 9 Tangent of a Sum

$$\tan(\pi/3 + \pi/4) = (\sqrt{3} + 1) \div (1 - \sqrt{3} \cdot 1) = (\sqrt{3} + 1)/(1 - \sqrt{3}).$$

Multiply top and bottom by $(1 + \sqrt{3})$: $(\sqrt{3} + 1)^2/(1 - 3) = (4 + 2\sqrt{3})/(-2) = -2 - \sqrt{3} \approx -3.732$.

Problem 10 Sum of Two Angles from Given Values

$\cos A = 15/17$ and $\sin B = 4/5$.

$$\sin(A + B) = (8/17)(-3/5) + (15/17)(4/5) = (-24 + 60)/85 = \mathbf{36/85}.$$

$$\cos(A + B) = (15/17)(-3/5) - (8/17)(4/5) = (-45 - 32)/85 = \mathbf{-77/85}.$$
 Check: $36^2 + 77^2 = 85^2 \checkmark$.

Problem 11 Double Angle from Cosine

$\sin \theta = 24/25$ (positive in Quadrant II).

$$\sin 2\theta = 2(24/25)(-7/25) = \mathbf{-336/625}.$$

$$\cos 2\theta = 49/625 - 576/625 = \mathbf{-527/625}.$$

Problem 12 Double Angle from Tangent

The reference triangle is 3-4-5, with $\sin \theta = -3/5$ and $\cos \theta = 4/5$.

$$\sin 2\theta = 2(-3/5)(4/5) = \mathbf{-24/25}; \cos 2\theta = 16/25 - 9/25 = \mathbf{7/25}.$$

$$\tan 2\theta = -24/7.$$
 Check with the formula: $2(-3/4) \div (1 - 9/16) = -24/7 \checkmark$.

Problem 13 Verify a Double-Angle Quotient

Use $1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 2\sin^2 x$ and $\sin 2x = 2 \sin x \cos x$.

The fraction is $2\sin^2 x / (2 \sin x \cos x) = \sin x / \cos x = \mathbf{\tan x}$. ■

Problem 14 Simplify to a Constant

Expand: $\sin x \cos x \cdot \sin x / \cos x + \sin x \cos x \cdot \cos x / \sin x = \sin^2 x + \cos^2 x = \mathbf{1}$.

Problem 15 A Longer Verification

Common denominator $\sin x(1 + \cos x)$: the numerator is $\sin^2 x + (1 + \cos x)^2 = \sin^2 x + 1 + 2\cos x + \cos^2 x = 2 + 2\cos x = 2(1 + \cos x)$.

The fraction becomes $2(1 + \cos x) / (\sin x(1 + \cos x)) = 2 / \sin x = \mathbf{2 \csc x}$. ■

Problem 16 Angle of Elevation of a Treetop

$$\tan(\alpha + \beta) = (1/4 + 3/5) \div (1 - (1/4)(3/5)) = (17/20) \div (17/20) = 1.$$

Since $\tan 45^\circ = 1$ and the angle is acute, the treetop's elevation is **45°** .

Problem 17 Range of a Launched Ball

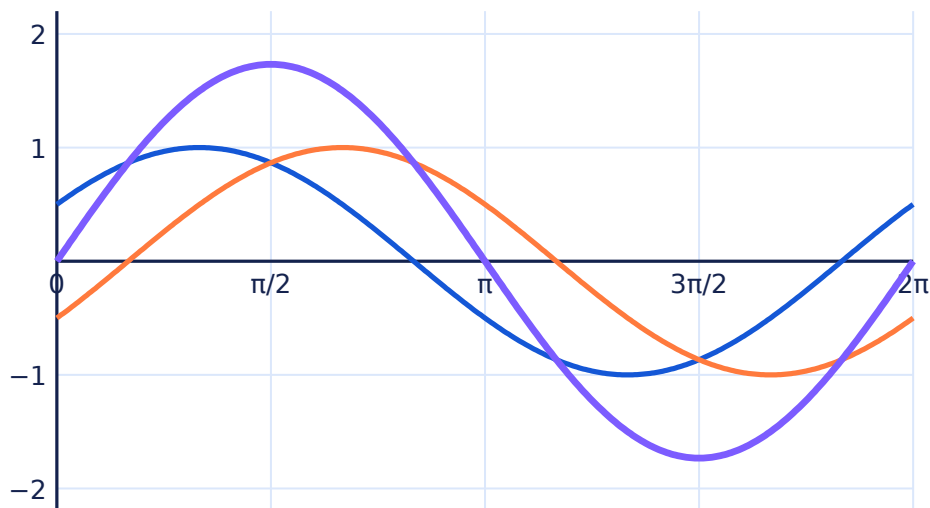
$\cos \theta = 0.8$, so $\sin 2\theta = 2(0.6)(0.8) = 0.96$. The factor $60^2/32 = 112.5$, so $R = 112.5 \times 0.96 =$ **108 feet**.

At 45° , $\sin 90^\circ = 1$ and $R =$ **112.5 feet**, only 4.5 feet farther.

Problem 18 Interfering Sound Waves

$$\sin(x + \pi/6) = \sin x \cos(\pi/6) + \cos x \sin(\pi/6) \text{ and } \sin(x - \pi/6) = \sin x \cos(\pi/6) - \cos x \sin(\pi/6).$$

Adding, the $\cos x$ terms cancel: $2 \sin x \cos(\pi/6) = 2 \sin x (\sqrt{3}/2) = \sqrt{3} \sin x$.



The sum has amplitude $\sqrt{3} \approx 1.73$.

Problem 19 Solve $\cos 2x = \cos x$

$$2\cos^2 x - 1 = \cos x, \text{ so } 2\cos^2 x - \cos x - 1 = 0 \text{ and } (2\cos x + 1)(\cos x - 1) = 0.$$

$\cos x = 1$ gives $x = 0$; $\cos x = -1/2$ gives $2\pi/3$ and $4\pi/3$. **$x = 0, 2\pi/3, 4\pi/3$** .

Problem 20 Solve $\sin 2x = \sin x$

$$2 \sin x \cos x - \sin x = 0, \text{ so } \sin x(2\cos x - 1) = 0.$$

$\sin x = 0$ gives $x = 0, \pi$; $\cos x = 1/2$ gives $\pi/3, 5\pi/3$. **$x = 0, \pi/3, \pi, 5\pi/3$.**

Dividing by $\sin x$ would lose the solutions 0 and π , where $\sin x = 0$.