

# Proving and Using Trigonometric Identities

Grade 12 – Precalculus math · Worksheet · Standards: F-TF.C.8, F-TF.C.9

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

These 20 problems cover the reciprocal, quotient and Pythagorean identities, verifying identities, sum and difference formulas, and double-angle formulas. They train standards F-TF.C.8 and F-TF.C.9. The last six problems apply the formulas to a treetop angle, a launched ball, interfering waves and two equations that need factoring.

**Standards:** F-TF.C.8, F-TF.C.9 · **Level:** Grade 12 Precalculus

## Problem 1 Reciprocal and Quotient Values (easy)

Suppose  $\sin \theta = 5/13$  and  $\cos \theta = 12/13$ . Find  $\tan \theta$ ,  $\csc \theta$ ,  $\sec \theta$  and  $\cot \theta$ .

## Problem 2 Simplify with Pythagorean Identities (easy)

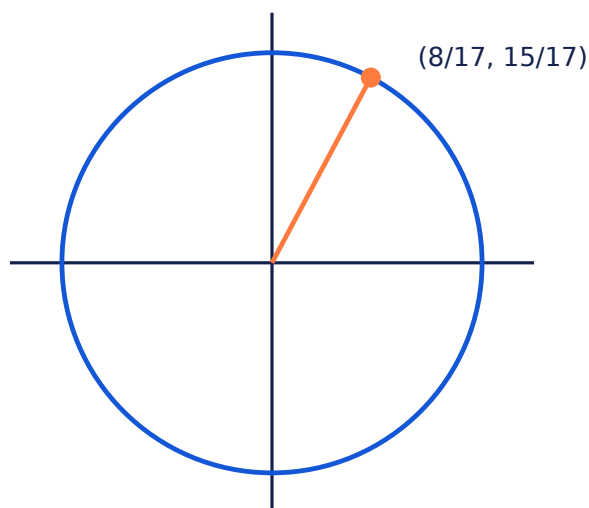
Simplify each expression. (a)  $5 - 5\sin^2 x$  (b)  $3\sec^2 x - 3$  (c)  $\csc^2 x - \cot^2 x$  (d)  $\cos x \tan x$

## Problem 3 Find Sine and Tangent from Cosine (easy)

Angle  $\theta$  is in Quadrant IV and  $\cos \theta = 5/13$ . Find  $\sin \theta$  and  $\tan \theta$ .

## Problem 4 Check a Point on the Unit Circle (easy)

Show that  $(8/17, 15/17)$  lies on the unit circle, then give  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$  and  $\sec \theta$  for the angle in standard position through this point.



The point  $(8/17, 15/17)$  on the unit circle.

### Problem 5 Identity or Not? (easy)

(a) Show that  $\sin^2 x + \cos^2 x = 1$  is true for  $x = \pi/6$ . (b) Show that  $\sin(x + y) = \sin x + \sin y$  is false for  $x = y = \pi/6$ .

### Problem 6 Exact Value with a Difference Formula (easy)

Use  $\cos(A - B)$  to find the exact value of  $\cos 15^\circ$ , using  $15^\circ = 45^\circ - 30^\circ$ .

### Problem 7 Verify a Quotient Identity (medium)

Verify that  $\sec x - \cos x = \sin x \tan x$ .

### Problem 8 Verify with Reciprocals (medium)

Verify that  $\csc x - \sin x = \cos x \cot x$ .

### Problem 9 Tangent of a Sum (medium)

Find the exact value of  $\tan(7\pi/12)$  using  $7\pi/12 = \pi/3 + \pi/4$ .

### Problem 10 Sum of Two Angles from Given Values (medium)

Angle  $A$  is in Quadrant I with  $\sin A = 8/17$ . Angle  $B$  is in Quadrant II with  $\cos B = -3/5$ . Find  $\sin(A + B)$  and  $\cos(A + B)$ .

**Problem 11 Double Angle from Cosine** (medium)

Angle  $\theta$  is in Quadrant II with  $\cos \theta = -7/25$ . Find  $\sin 2\theta$  and  $\cos 2\theta$ .

**Problem 12 Double Angle from Tangent** (medium)

Angle  $\theta$  is in Quadrant IV with  $\tan \theta = -3/4$ . Find  $\sin 2\theta$ ,  $\cos 2\theta$  and  $\tan 2\theta$ .

**Problem 13 Verify a Double-Angle Quotient** (medium)

Verify that  $\frac{1 - \cos 2x}{\sin 2x} = \tan x$ .

**Problem 14 Simplify to a Constant** (medium)

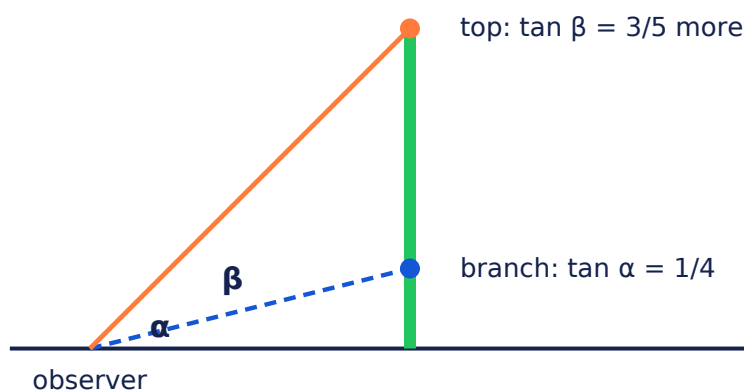
Simplify  $\sin x \cos x (\tan x + \cot x)$ .

**Problem 15 A Longer Verification** (hard)

Verify that  $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \csc x$ .

**Problem 16 Angle of Elevation of a Treetop** (hard)

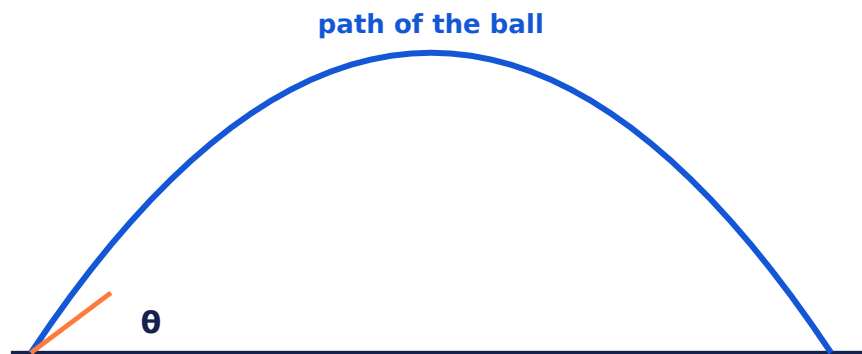
From a spot on level ground, a branch is seen at angle  $\alpha$  with  $\tan \alpha = 1/4$ . The treetop is seen at an angle  $\alpha + \beta$  where  $\tan \beta = 3/5$ . Find  $\tan(\alpha + \beta)$  and the angle of elevation of the treetop.



Two lines of sight to a tree.

**Problem 17 Range of a Launched Ball** (hard)

A ball launched at  $v = 60$  feet per second and angle  $\theta$  travels a horizontal distance  $R = (v^2/32) \sin 2\theta$  feet. If  $\sin \theta = 0.6$  and  $\theta$  is acute, find  $R$ . Compare with a  $45^\circ$  launch.

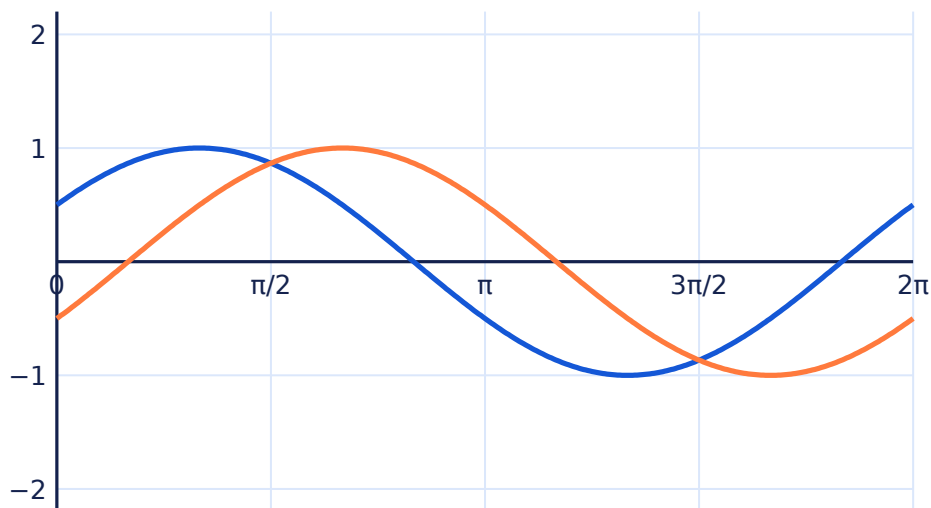


$$\text{range } R = (v^2/32) \sin 2\theta \text{ feet}$$

*A launched ball and its range.*

### Problem 18 Interfering Sound Waves (hard)

Two speakers produce waves modeled by  $y_1 = \sin(x + \pi/6)$  and  $y_2 = \sin(x - \pi/6)$ . Use the sum and difference formulas to write  $y_1 + y_2$  as a single multiple of  $\sin x$ .



*The two waves.*

### Problem 19 Solve $\cos 2x = \cos x$ (hard)

Solve  $\cos 2x = \cos x$  on  $[0, 2\pi)$ . Use  $\cos 2x = 2\cos^2 x - 1$ .

### Problem 20 Solve $\sin 2x = \sin x$ (hard)

Solve  $\sin 2x = \sin x$  on  $[0, 2\pi)$ , and explain why dividing by  $\sin x$  is not allowed.