

Radians, Arc Length and the Unit Circle

Grade 12 – Precalculus math · Test Answer Key · Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4

Each solution shows the work with a scoring note. Award method points for the reference angle, the sign and the final value separately, so one sign slip costs only a single point.

Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · **Level:** Grade 12 Precalculus

Question 1 Convert Angle Measures (2 points)

(a) $315\pi/180 = 7\pi/4$.

(b) $7\pi/18 \cdot 180/\pi = 70^\circ$.

Scoring: 1 point per conversion.

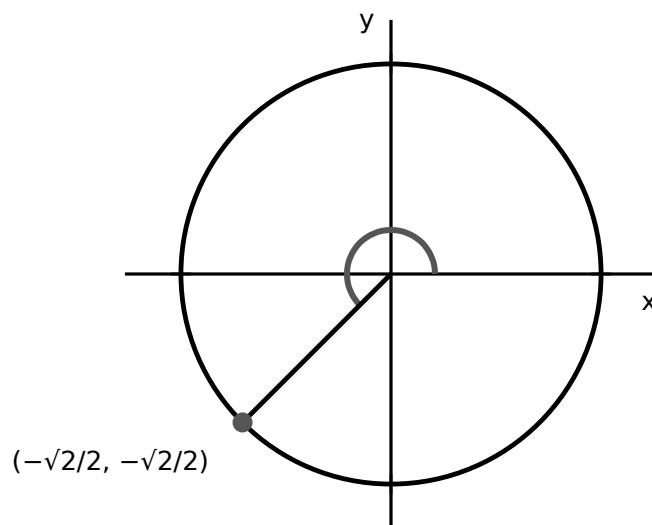
Question 2 Sprinkler Arc (2 points)

a. $s = 18 \cdot 5\pi/6 = 15\pi \approx 47.1$ feet.

b. $\theta = 30/18 = 5/3$ rad = $5/3 \cdot 180/\pi \approx 95.5^\circ$.

Scoring: 1 point per part.

Question 3 Point on the Unit Circle (2 points)



Angle $\theta = 5\pi/4$.

- a. The reference angle is $\pi/4$ and both coordinates are negative: $(-\sqrt{2}/2, -\sqrt{2}/2)$.
b. $\tan \theta = \sin \div \cos = 1$.

Scoring: 1 point per part.

Question 4 Exact Trigonometric Values (3 points)

- (a) Quadrant II, reference $\pi/4$: $-\sqrt{2}/2$.
(b) $\sin(-7\pi/6) = -\sin(7\pi/6) = -(-1/2) = 1/2$.
(c) Quadrant IV, reference $\pi/3$: $(-\sqrt{3}/2) \div (1/2) = -\sqrt{3}$.

Scoring: 1 point per value.

Question 5 Find Cosine and Tangent (2 points)

$\cos^2 \theta = 1 - 25/169 = 144/169$. In Quadrant III cosine is negative, so $\cos \theta = -12/13$.

$\tan \theta = (-5/13) \div (-12/13) = 5/12$.

Scoring: 1 point for $\cos \theta$ with the correct sign; 1 point for $\tan \theta$.

Question 6 Reference Angle and Sine (2 points)

$17\pi/6 - 2\pi = 5\pi/6$, which is in Quadrant II.

- (a) Reference angle: $\pi - 5\pi/6 = \pi/6$.
(b) Sine is positive in Quadrant II: $1/2$.

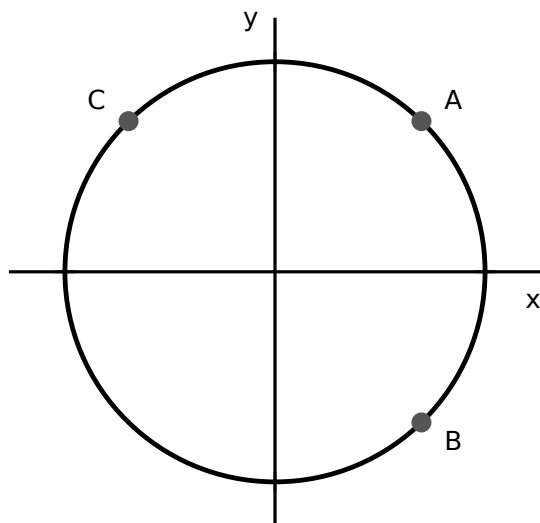
Scoring: 1 point per part.

Question 7 Tire Rotation (3 points)

- a. $\omega = 420 \cdot 2\pi = 840\pi \approx 2,638.9$ rad/min.
b. $v = 15 \cdot 840\pi = 12,600\pi \approx 39,584$ in/min.
c. $39,584 \times 60 \div 63,360 \approx 37.5$ mph.

Scoring: 1 point per part.

Question 8 Symmetry of Marked Points (2 points)



Points A, B and C on the unit circle.

- a. Cosine is even and sine is odd: $B = (\cos 0.8, -\sin 0.8) \approx \mathbf{(0.697, -0.717)}$.
- b. $\cos(\pi - 0.8) = -\cos 0.8$ and $\sin(\pi - 0.8) = \sin 0.8$, so $C \approx \mathbf{(-0.697, 0.717)}$.

Scoring: 1 point per point, both coordinates required.

Question 9 Particle Motion (2 points)

- (a) $\theta = 7\pi/3 = 2\pi + \pi/3$, so the position is $\mathbf{(1/2, \sqrt{3}/2)}$.
- (b) The point $(-1, 0)$ is at angle π : $(\pi/3)t = \pi$ gives $\mathbf{t = 3 \text{ seconds}}$.

Scoring: 1 point per part.