

Radians, Arc Length and the Unit Circle

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4

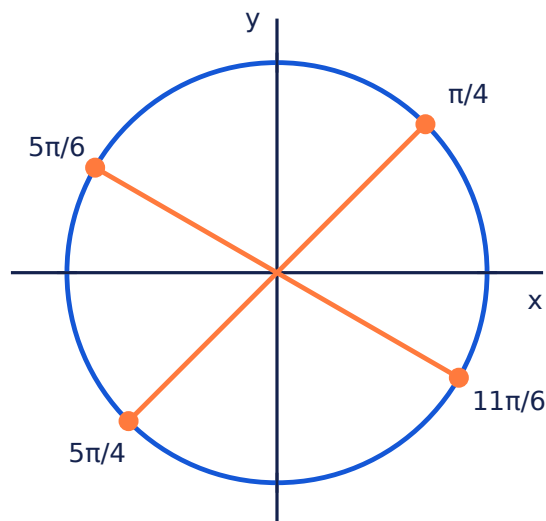
Each answer shows the conversion or reference-angle step and gives the exact value in bold, with a decimal where a measurement is involved. Redraw the angle on a unit circle to confirm every sign.

Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · **Level:** Grade 12 Precalculus

Problem 1 Degrees to Radians

Multiply by $\pi/180$.

(a) $45\pi/180 = \pi/4$. (b) $150\pi/180 = 5\pi/6$. (c) $225\pi/180 = 5\pi/4$. (d) $330\pi/180 = 11\pi/6$.



Four angles drawn on the unit circle.

Problem 2 Radians to Degrees

Multiply by $180/\pi$.

(a) **120°** . (b) **315°** . (c) $3 \cdot 18^\circ = \mathbf{54^\circ}$. (d) $5 \cdot 15^\circ = \mathbf{75^\circ}$.

Problem 3 Arc Length Basics

Use $s = r\theta$.

(a) $12 \cdot 5\pi/6 = 10\pi \approx 31.42$ inches.

(b) $9 \cdot \pi/3 = 3\pi \approx 9.42$ feet.

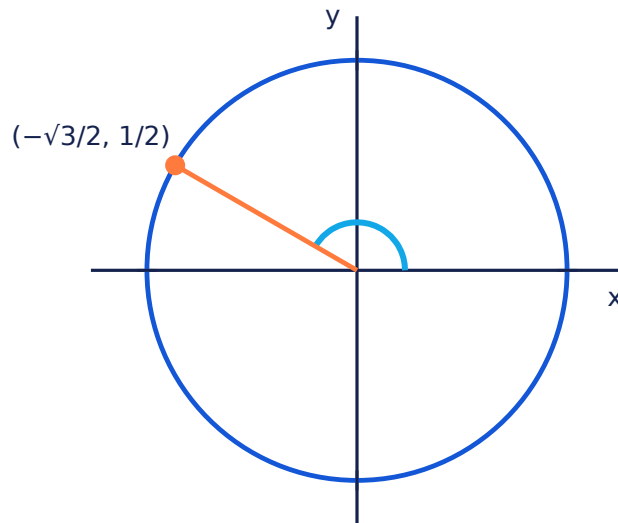
Problem 4 Special Angle Values

(a) $\sqrt{3}/2$. (b) $\sqrt{3}/2$. (c) $\sin \div \cos = (\sqrt{2}/2) \div (\sqrt{2}/2) = 1$. (d) The point is (0, 1), so 0.

Problem 5 Coordinates from the Unit Circle

Quadrant II. Reference angle: $\pi - 5\pi/6 = \pi/6$.

At $\pi/6$ the point is $(\sqrt{3}/2, 1/2)$; in Quadrant II the x-coordinate is negative: $(-\sqrt{3}/2, 1/2)$.



The angle drawn in the figure.

Problem 6 Quadrants and Reference Angles

(a) $\pi < 4\pi/3 < 3\pi/2$: Quadrant III, reference $4\pi/3 - \pi = \pi/3$.

(b) Quadrant IV, reference $2\pi - 7\pi/4 = \pi/4$.

(c) Quadrant II, reference $\pi - 5\pi/6 = \pi/6$.

Problem 7 Exact Values in Every Quadrant

(a) Quadrant III, reference $\pi/4$: $-\sqrt{2}/2$.

(b) Quadrant II, reference $\pi/6$: $-\sqrt{3}/2$.

(c) Quadrant III, $\tan = (-1/2) \div (-\sqrt{3}/2) = 1/\sqrt{3} = \sqrt{3}/3$.

(d) Quadrant IV, reference $\pi/6$: $\sqrt{3}/2$.

Problem 8 Negative and Large Angles

(a) Cosine is even: $\cos(\pi/3) = 1/2$.

(b) Sine is odd: $-\sin(3\pi/4) = -\sqrt{2}/2$.

(c) $13\pi/6 - 2\pi = \pi/6$, so $1/2$.

(d) $17\pi/4 - 4\pi = \pi/4$, so $\sqrt{2}/2$.

Problem 9 Even and Odd Relationships

(a) Odd: $-a$. (b) Even: b .

(c) $9\pi/8 = \pi + \pi/8$ is in Quadrant III, so sine is negative: $-a$.

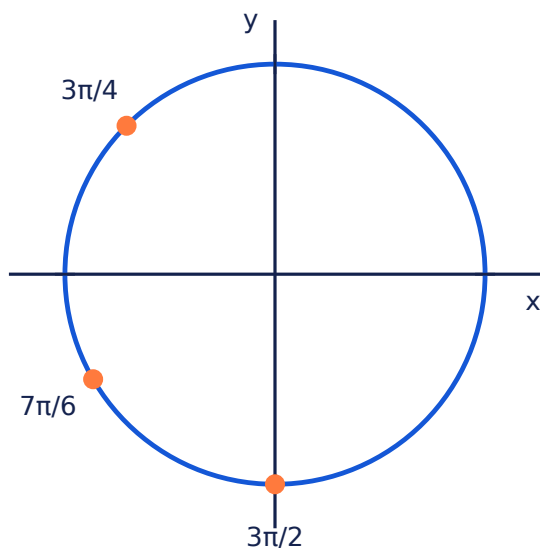
(d) $7\pi/8 = \pi - \pi/8$ is in Quadrant II, so cosine is negative: $-b$.

Problem 10 Find the Angle from a Point

(a) Quadrant III with reference $\pi/6$: $7\pi/6$.

(b) The bottom of the circle: $3\pi/2$.

(c) Quadrant II with reference $\pi/4$: $3\pi/4$. All angles: $3\pi/4 + 2\pi k$ for any integer k .



The three points of the problem.

Problem 11 Find Cosine and Tangent from Sine

On the unit circle $x^2 + y^2 = 1$, so $\cos^2 \theta = 1 - 9/25 = 16/25$. In Quadrant II cosine is negative:
 $\cos \theta = -4/5$.

$$\tan \theta = (3/5) \div (-4/5) = -3/4.$$

Problem 12 Angle from Arc Length

$$\theta = s/r = 15/6 = 2.5 \text{ radians.}$$

$$\text{In degrees: } 2.5 \cdot 180/\pi \approx 143.2^\circ.$$

Problem 13 Wheel Speeds

One revolution is 2π radians: $\omega = 240 \cdot 2\pi = 480\pi \approx 1,508.0 \text{ rad/min.}$

$$v = r\omega = 14 \cdot 480\pi = 6,720\pi \approx 21,112 \text{ in/min. Then } 21,112 \times 60 \div 63,360 \approx 20.0 \text{ mph.}$$

Problem 14 Coterminal Angles

Add or subtract 2π : $5\pi/6 + 2\pi = 17\pi/6$; $5\pi/6 + 4\pi = 29\pi/6$; $5\pi/6 - 2\pi = -7\pi/6$.

Problem 15 Tangent Values

$$(a) (\sqrt{3}/2) \div (-1/2) = -\sqrt{3}. (b) (-\sqrt{2}/2) \div (-\sqrt{2}/2) = 1.$$

$$(c) \text{Tangent is odd: } -\sqrt{3}/3. (d) \cos(3\pi/2) = 0, \text{ so undefined.}$$

Problem 16 Ferris Wheel Travel

One turn is 2π radians in 240 seconds: $\omega = 2\pi/240 = \pi/120 \text{ rad/s.}$

$$\text{In 50 seconds, } \theta = 50\pi/120 = 5\pi/12, \text{ so } s = 25 \cdot 5\pi/12 = 125\pi/12 \approx 32.72 \text{ feet.}$$

$$v = r\omega = 25\pi/120 = 5\pi/24 \approx 0.654 \text{ ft/s.}$$

Problem 17 Minute Hand Tip

The hand turns 2π in 60 minutes, so in 20 minutes $\theta = 2\pi/3$ and $s = 9 \cdot 2\pi/3 = 6\pi \approx 18.85$ inches.

In 50 minutes $\theta = 5\pi/3$ and $s = 9 \cdot 5\pi/3 = \mathbf{15\pi \approx 47.12 \text{ inches}}$.

Problem 18 Bicycle Wheel Rotation

Speed: $12 \text{ mi/h} = 12 \cdot 63,360 \div 60 = 12,672 \text{ in/min}$. The radius is 13.5 in.

$$\omega = v/r = 12,672 \div 13.5 \approx \mathbf{938.7 \text{ rad/min}}.$$

Divide by 2π : $\approx \mathbf{149.4 \text{ rpm}}$.

Problem 19 Lighthouse Beam

$$\omega = 2 \cdot 2\pi = 4\pi \text{ rad/min, so } v = r\omega = 3 \cdot 4\pi = \mathbf{12\pi \approx 37.7 \text{ mi/min}}.$$

$$30^\circ = \pi/6 \text{ rad, so } s = 3 \cdot \pi/6 = \mathbf{\pi/2 \approx 1.57 \text{ miles}}.$$

Problem 20 A Particle on the Unit Circle

After 2 s: $\theta = 3\pi/2$, position $\mathbf{(0, -1)}$.

After 5 s: $\theta = 15\pi/4 - 2\pi = 7\pi/4$, position $\mathbf{(\sqrt{2}/2, -\sqrt{2}/2)}$.

First return when $(3\pi/4)t = 2\pi$, so $t = \mathbf{8/3 \text{ seconds}}$.